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## High order thinking skills: can it arise when a prospective teacher solves a controversial mathematics problem?

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**Abstract.** High order thinking skills are important for prospective mathematics teachers. This study aims to describe whether High Order Thinking Skills (HOTS) can appear when prospective teachers solve controversial math problems. This research has been conducted using descriptive qualitative research. The instruments that have been used are controversial math test questions and interviews. The research subjects used were 150 prospective mathematics teachers, and three people who experienced controversy were taken to find out their HOTS. The results showed that from the three subjects, information was obtained that analysis and evaluation had emerged when solving controversial problems, but only one subject appeared for creating. Suggestions for the next researcher is to combine critical thinking and HOTS to bring up aspects of creating.

### 1. Introduction

Controversial issues are important in everyday life [1-6]. Controversial issues are needed in a learning process in which there is a process of discussion and argumentation[1]. Controversial issues make the learning process more lively and active [1,7]. In the classroom, given a controversial problem, many new ideas emerge [8,9]. This shows that controversial issues are needed in everyday life.

Controversial math problems cause differences of opinion, raise curiosity, and make someone make an argument [6,7]. Research on controversial issues has been carried out by [6,7, 9–12]. The result of research [6] states that controversial issues can lead to conflicting discussions to make the learning process more active. This is done so that students are accustomed to solving different problems and are more active in discussing. The result of research [11-12] mentions that students' arguments when solving controversial problems can make them more critical and find new things.

Furthermore, [9] explains that controversial math problems are important in everyday life and lead to prospective teachers' critical thinking. With critical thinking, so that prospective mathematics teachers need to be accustomed to solving controversial problems. Therefore, a preliminary study is necessary to find out how the results of prospective mathematics teachers after solving controversial problems.

The researcher's preliminary study was conducted to determine how the prospective teacher solved a controversial math problem. The results of the initial study obtained by the researchers received several facts, namely: a) prospective teachers are not used to solving controversial math problems, b) after solving controversial problems, prospective teachers carry out the process of analyzing and evaluating their work, c) after solving controversial problems, prospective teachers have ideas new to complete other similar.



High Order Thinking Skills (HOTS) is a process of classifying something complex into several parts, detecting relationships, combining new information, and using all previous levels to evaluate or make judgments [13–17]. In HOTS, there is a process to identify a concept, see the relationship between the idea and the information previously obtained, and then assess the analysis results [18–20]. This is in line with the preliminary study results, which states that there is a process of analysis, evaluation, and creating new things. Therefore, this study wants to see how the HOTS of prospective mathematics teachers is after solving a controversial problem.

Research on HOTS is divided into three broad categories: instruments based on HOTS, learning models that generate HOTS, and analysis of students' HOTS processes in solving problems. This study collaborates between two categories, namely by using controversial issues, then analyzing using HOTS. Research on HOTS that has been carried out includes: [13,15, 17, 18, 21–23]. From these studies, no one has used controversial mathematical problems and conducted an analysis using HOTS. Therefore, this study aims to describe whether HOTS can appear after prospective mathematics teachers solve controversial problems.

## 2. Research Method

### 2.1. Research Approach

This research has been conducted using a descriptive qualitative research. A qualitative approach is used to find out in-depth information about the high-order thinking skills of prospective teachers after solving a controversial math problem. The characteristics of natural background and the data collected in words and pictures are also the characteristics of the qualitative research in this study. Data were collected using purposive sampling.

### 2.2. Research Instruments

The instruments that have been used are controversial tests, observations, and interviews to determine the higher-order thinking skills of prospective teachers. For example, the following is a controversial question given to 150 teacher candidates. The controversial question given is done for 20 minutes. The following are controversial questions given to prospective teachers.

Figure 1

Figure 2

Figure 3

Figure 4

It is known that Figure 1 is an isosceles right triangle with the length of the hypotenuse being  $c$ , and the size of the right side is  $b = a$ . Based on the Pythagorean Theorem, we get the equation  $a^2 + b^2 = c^2$ . Because  $b = a$  then obtained  $a^2 + a^2 = 2a^2 = c^2$ . So  $c = \sqrt{2}a$ .

Consider Figure 2, a ladder made on a sloping side, where the height and width are  $b$ . More and more stairs are made, with the height and width of each step being the same, as can be seen in Figures 3 and 4. Look at Figure 4, and it can be seen that the length of the hypotenuse  $c$  is formed from stairs whose height and width are  $\frac{a}{n}$  and  $\frac{b}{n}$ ,  $n \in \mathbb{N}$ . So the length of the hypotenuse  $c$  is  $\left(n \times \frac{a}{n}\right) + \left(n \times \frac{b}{n}\right)$ . Because it is known that  $a = b$ , then the length of the hypotenuse  $c$  is  $\left(n \times \frac{a}{n}\right) + \left(n \times \frac{a}{n}\right) = \frac{an}{n} + \frac{an}{n} = \frac{2an}{n} = 2a$ . It means that the value of  $c = 2a$ .

1. Do the above results contradict the results  $c = \sqrt{2}a$  obtained from the Pythagorean theorem? Explain your answer!
2. As a teacher, how do you convey to students when you encounter these problems?

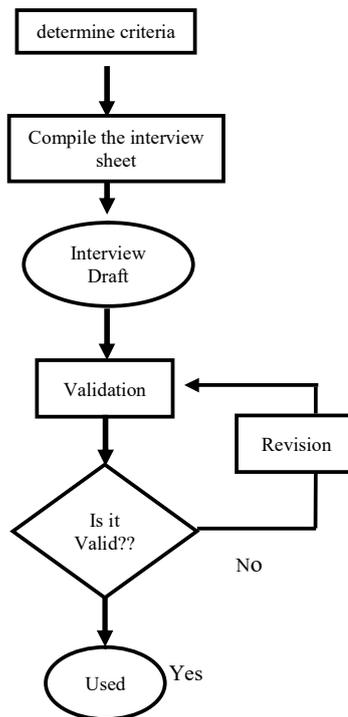
Figure 1. Mathematics Controversial Problems.

Furthermore, in Table 1, a grid of interview guidelines is used to determine the prospective teacher's high-order thinking skills.

**Table 1.** Interview Guide Grid.

Category	Aspect	Facts That Need To Be Revealed
Analysis	Differentiate	Distinguish what is relevant and not from the material presented.
	Organize	Determine the appropriate concept and can be used in the material presented.
	Connect	Determine the point of view of the material presented.
Evaluation	Check	Detect inconsistencies in a process or product. Detect the effectiveness of the procedures being implemented
	Discuss	Detect inconsistencies between products and external criteria. Detect the conformity of the procedure with the given problem
Create	Awaken	Propose a hypothesis
	Plan	Designing procedures to solve problems
	produce	Creating results or products

Figure 2 below is the flow of the preparation of the interview sheet.



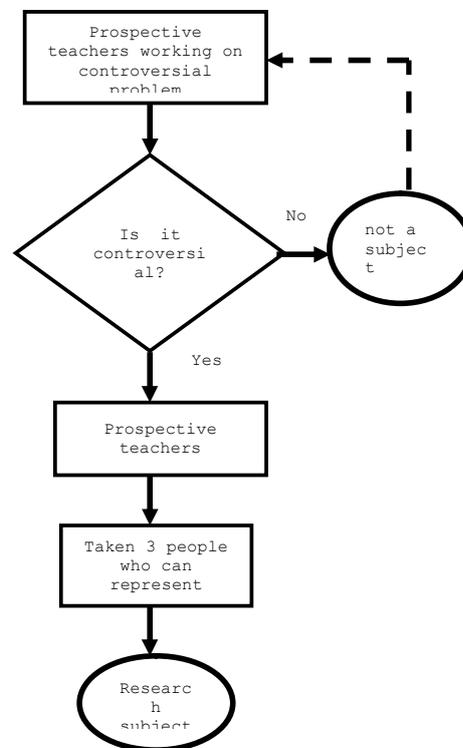
**Figure 2.** Flow of Preparation of Interview Sheet.

### 2.3. Data Analysis

Data analysis carried out in this study is to collect data, perform data reduction, present data in the form of images or tables, and conclude the process of collecting data through tests and interviews. A prospective mathematics teacher solves a given controversial problem, and then the work is corrected. By using data reduction, three subjects were taken from 150 prospective mathematics teachers who solved controversial problems. The three research subjects were interviewed. The results of interviews and the work of research subjects were analyzed. The results of the analysis are presented in the form of images or tables. The final step is to conclude the results of the study to answer the research problem formulation.

### 2.4. Determination of Research Subjects

Figure 3 below is the determination of the research subject.

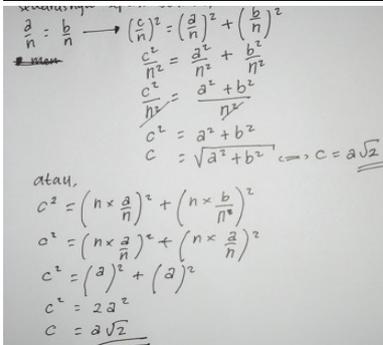
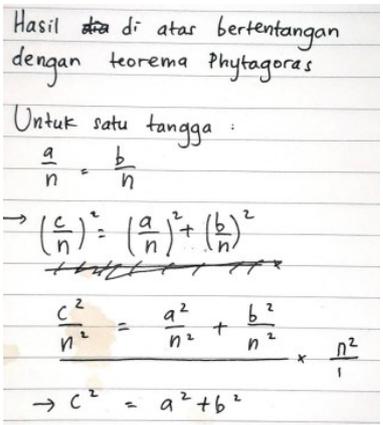
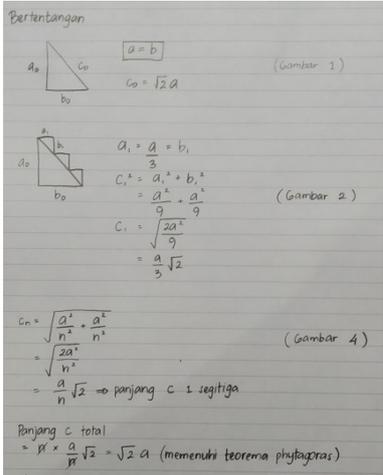


**Figure 3.** Participant Selection Flow.

## 3. Research Results and Discussion

The results of the work of research subjects are classified into three parts. Each part is taken one subject, which is considered to represent the group. The following Table 2 is the result of the work of the subject that has been classified.

**Table 2.** Research Subjects' Work Results.

	Working result	Characteristic
Subject 1	 <p>Handwritten mathematical derivation for Subject 1. It starts with the equation <math>\frac{a}{n} = \frac{b}{n}</math> and derives <math>c = a\sqrt{2}</math> through several steps, including squaring both sides and simplifying.</p>	Using one point of view, namely the Pythagorean theorem, to solve controversial mathematical problems
Subject 2	 <p>Handwritten mathematical derivation for Subject 2. It starts with the text 'Hasil di atas bertentangan dengan teorema Pythagoras' and 'Untuk satu tangga: <math>\frac{a}{n} = \frac{b}{n}</math>'. It then derives <math>c^2 = a^2 + b^2</math> through several steps, including squaring both sides and simplifying.</p>	Using the Pythagorean theorem and substituting to the second method on the given problem
Subject 3	 <p>Handwritten mathematical derivation for Subject 3. It starts with a geometric diagram labeled 'Pertentangan' and shows a right-angled triangle with sides <math>a_0</math>, <math>b_0</math>, and <math>c_0</math>. It then derives <math>c = a\sqrt{2}</math> through several steps, including squaring both sides and simplifying.</p>	Knowing the point of view using the Pythagorean theorem and using non-Euclidean geometry, but has not been able to solve the controversial problem given

**3.1. Analysis**

In the analysis aspect, it be seen whether the research subject can distinguish what is relevant from the material presented, determine the appropriate concept, and determine the point of view of the material presented.

**Table 3.** Subject's Work Results on Aspect Category of Analysis.

Category	Aspect	Facts That Need To Be Revealed	Subject 1 (S1)	Subject 2 (S2)	Subject 3 (S3)
Analysis	Differentiate	Distinguish what is relevant and not from the material presented.	S1 has not been able to distinguish what is relevant and not from the material presented.	S2 has not been able to distinguish which is relevant and not from the given problem because both solutions must use Euclid's geometric concepts.	S3 can be seen to determine which ones are relevant and not only from one answer, namely the first method. In contrast, the process is not explained by S3 either in written answers or interviews.
	Organize	Determine the appropriate concept and can be used in the material presented.	S1 has not been able to determine the appropriate concept and can be used in the material presented.	S2 has not been able to determine the appropriate concept and can be used in the material presented.	S3 determines the appropriate concept when completing the first method but has not discussed the second method.
	Connect	Determine the point of view of the material presented.	S1 has not been able to determine the point of view of the material presented	S2 has not been able to determine the point of view of the material presented.	S2 has not been able to determine the point of view of the material presented.

In the analysis category, it can be seen that for the distinguishing aspect, only S3 can determine which ones are relevant and which are not. The process of determining the relevant material has been researched by [24]. The results of his research stated that the evaluation aspect is the results obtained easily by students and prospective teachers. Bloom's taxonomy level one or known as the analysis stage is the basic stage that most prospective teachers can master [13,25]. However, when determining the point of view of the material presented, prospective teachers are fooled when solving controversial math problems. Controversial mathematics problems can be an alternative to raise problems that increase the critical thinking of prospective teachers [9].

3.2. Evaluation

In the evaluation aspect, some facts need to be revealed: detecting inconsistencies in a problem-solving process and seeing the procedure's suitability with the given problem.

**Table 4.** Subject's Work Results on Aspect Category of Evaluation.

Category	Aspect	Facts That Need To Be Revealed	Subject 1 (S1)	Subject 2 (S2)	Subject 3 (S3)
Check		Detect inconsistencies in a process or product.	S1 can be seen that it has been working consistently, namely by substituting the		
		Detect the		S2 can also be seen	S3 can detect

Category	Aspect	Facts That Need To Be Revealed	Subject 1 (S1)	Subject 2 (S2)	Subject 3 (S3)
Evaluation		effectiveness of the procedures being implemented	Pythagorean theorem into a known equation.	that what is written is also consistent.	effectiveness in a process or product.
	Discuss	Detect inconsistencies between products and external criteria. Detect the conformity of the procedure with the given problem	S1 has not mentioned the non-Euclidean geometries used to solve the second way on a given controversial problem.	S2 has been able to detect the suitability of the procedure with the given problem.	S3 can detect the suitability of the procedure with the given problem.

In the evaluation section, there is a checking and discussion process [14,16, 22, 26]. During the checking process, the prospective teacher detects consistency. In addition, prospective teachers also detect the effectiveness of the procedures used. The results of research [23] stated that by checking the results of their work, prospective teachers became accustomed to solving other similar problems. In the discussion process, prospective teachers detect the suitability of the procedures used. This discussion activity can be carried out for the process of self-reflection or with other friends after solving problems [27–30].

3.3. Creates

Some facts need to be revealed in creating, namely proposing hypotheses, designing procedures to solve problems, and create results.

**Table 5.** Subject's Work Results on Aspect Category of Create.

Category	Aspect	Facts That Need To Be Revealed	Subject 1 (S1)	Subject 2 (S2)	Subject 3 (S3)
	Awaken	Propose a hypothesis	S1 has not been able to submit an idea.	S2 can propose a hypothesis. The hypotheses are: understand the problem, understand the material and solve it effectively.	S3 can propose a hypothesis.
	Plan	Designing procedures to solve problems	S1 has not been able to design procedures to solve new problems.	S2, it was obtained information that the planning process delivered by S2 tipped for solving problems, not procedural problem solving	S3 can design procedures to solve new problems

Category	Aspect	Facts That Need To Be Revealed	Subject 1 (S1)	Subject 2 (S2)	Subject 3 (S3)
Create	Produce	Creating results or products	S1 has not been able to create results or products.	S2 hasn't come up with any ideas to solve the problem. Here is a snippet.	S3 has explained the idea of solving the problem, but he still doubts whether the concept is new.

The aspect of creating is one of the aspects in which there are creating, planning and producing. The weakness of prospective mathematics teachers is that they are not used to discovering new things [31,32]. The results of research [13, 23, 33] stated that prospective teachers have been able to design procedures, but have not been able to create new results.

#### 4. Conclusions and Suggestions

The results of the study state that in the aspect of analysis, only S1 can distinguish between which are relevant and not from the material presented. S3 can determine the concept that appropriate when completing the first method but has not discussed the second method. S3 can determine the point of view of the material presented. In the evaluation aspect, the three subjects were able to detect inconsistencies in the work process provided. Furthermore, in the evaluation aspect, the three subjects can be seen that they have been working consistently, namely by substituting the Pythagorean theorem into a known equation. Furthermore, in the aspect of creating, S2 and S3 can propose hypotheses when solving problems. But the three subjects have not been able to produce results or products.

Future research seeks to determine the appropriate collaboration between HOTS and the relevant learning model or method. The hope is that it can bring up the creative aspect of HOTS. In addition, the next researcher can design research instruments that can create results or products.

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