

A Literature Review Of Multimodal Semiotic Reasoning In Mathematics

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ABSTRACT

In learning mathematics in the 21st century and after the Covid-19 pandemic, a multimodal role in the process of drawing conclusions involving mathematical symbols or signs is very much needed. This process is called multimodal semiotic reasoning. This research aims to study the literature on multimodal semiotic reasoning research articles. Until April 2023, there were 32 articles on the keyword multimodal semiotic reasoning with the subject of mathematics in the Scopus, Science Direct, and ERIC databases. Furthermore, it is identified based on a) suitability of criteria; b) the similarity of the articles in the three databases. Finally obtained 24 articles that meet the requirements for further research. A literature review on multimodal semiotic reasoning is explored based on a) kinds of mathematical reasoning involving symbols and signs using multimodal, b) multimodal used in research, c) the most researched mathematical concepts and the mathematization process that occurs, d) the role of multimodal semiotic reasoning in mathematics education. The results showed that a) the most dominant type of mathematical reasoning involving objects, symbols, and signs using multimodal was Algorithmic Reasoning (AR), b) the most multimodal source was verbal answers, c) the concept of geometry through a process of vertical-horizontal mathematization was the most researched, and 4) the biggest role of multimodal semiotic reasoning for the benefit of knowledge of mathematics education. In conclusion, multimodal semiotic reasoning has had a major impact on mathematics education and research on this multimodal semiotic reasoning will continue to thrive in the future.

Keywords: reasoning, semiotic, multimodal

INTRODUCTION

The 21st-century mathematics learning and the post-covid-19 pandemic demand various changes in the learning process (Ramloud et al., 2021; Schleicher, 2020). The changes that occur require a review to accommodate the various potentials of students (Engelbrecht et al., 2023). Currently, audio-visual roles such as interview assignments, experiments, practicals, presentations, dialogues, and others are an important key to learning mathematics (Marshall & Conana, 2021; O'Halloran, 2015). Assignments are no longer in written form but go beyond them. 21st-century learning demands that assignments given by mathematics teachers can accommodate verbal, psychomotor, imagery, and artifacts produced by students in learning (Alyousef, 2016; Clark-Wilson et al., 2020). During the Covid-19 pandemic, answers other than written ones were an alternative for students to communicate their thoughts (Banzato & Coin, 2019; Marshall & Conana, 2021). In this case, multimodal roles (body movements, use of manipulatives, drawing actions, pictorial displays, verbal language, and use of written symbols) become important aspects of learning mathematics (Herakleioti & Pantidos, 2016). These important aspects provide benefits for uncovering the processes that occur in learning mathematics (Duijzer, Van den Heuvel-Panhuizen, Veldhuis, Doorman, et al., 2019; Solovieva et al., 2016).

In learning mathematics, students carry out thought processes to make decisions (Pasani, 2019). The process of thinking about something to make a decision is called reasoning (Llinares, 2018; Saleh et al., 2018). The reasoning is not limited to the thought process for making decisions, but decisions made also make sense by providing logical reasons (Gaudêncio, 2020). Logical reasons produced by students in learning mathematics are obtained through the stages of analyzing information, gathering evidence, drawing conclusions, and expressing opinions (Sheehan et al., 2019).

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In mathematics class, teachers involve more concrete objects, pictures, signs, or symbols to help students do mathematical reasoning (Fiantika et al., 2018; Maffia & Mariotti, 2020). The reasoning that students do by involving signs and symbols is used to get the best explanation based on the ability to remember facts that have been seen and observed by students (Reisberg, 2017; C. W. Suryaningrum & Ningtyas, 2019). Reasoning related to symbols or signs is called semiotic reasoning (Oizumi et al., 2018; Christine Wulandari Suryaningrum et al., 2020; Tytler et al., 2020). In this study, the process of making decisions related to objects, symbols, or signs through the integration of various sources originating from student writing, oral answers, hand-finger-arm movements (gestures), student facial expressions, pictures, concrete objects, and artifacts is used. students are called multimodal semiotic reasoning (Danielsson & Selander, 2021; Tytler et al., 2020).

Based on previous research, multimodal semiotic reasoning is an important aspect of understanding mathematics learning through the communication domain, with an emphasis on content issues, interpersonal aspects, and the role of communicative sources and artifacts (Nordin & Björklund Boistrup, 2018). A multimodal semiotic framework can be used to investigate signs of potential learning in students' multimodal representations when performing reasoning activities (S Kjällander et al., 2021). The use of multimodal semiotic reasoning contexts in textbooks (Daher & Abu Thabet, 2020), digital teaching materials (Bergvall & Dyrvoid, 2021), and learning processes (Espeland et al., 2018) are some evidence of the importance of semiotic reasoning in the process of learning mathematics. The question is, a) what kinds of mathematical reasoning involve symbols, and signs using multimodal? b) what multimodal is most used in research? c) what mathematical concepts are most researched and how does the process of mathematization occur? d) what is the role of multimodal semiotic reasoning in mathematics education?

This study aims to study the literature on research articles on multimodal semiotic reasoning. A search for articles on multimodal semiotic reasoning with the subject of mathematics was carried out in the Scopus, Science Direct, and ERIC databases. This research will explore based on a) kinds of mathematical reasoning involving symbols, and signs using multimodal, b) multimodal used in research, c) the most researched mathematical concepts and the mathematization processes that occur, and d) the role of semiotic reasoning multimodal in mathematics education.

METHOD

Research design

The main focus of this research is to explore research results that discuss multimodal semiotic reasoning in mathematics education. This literature review covers four aspects of the publication of multimodal semiotic reasoning based on a) kinds of mathematical reasoning involving symbols, and signs using multimodal, b) multimodal used in research, c) the most researched mathematical concepts and the mathematization processes that occur, d) the role of semiotic reasoning multimodal in mathematics education. The research was conducted by conducting an exploratory study of three journal databases, namely Scopus, Science Direct, and the Education Resources Information Center (ERIC) using the keyword multimodal semiotic reasoning with subject mathematics.

The article is focused on research results related to multimodal semiotic reasoning indexed by Scopus, Science Direct, and ERIC. The selection of the Scopus database is because the international journals listed in the Scopus database have a good reputation. In addition, the search for reputable international journals is also through Science Direct, which is the first online library for electronic (scientific) books and articles under the auspices of Elsevier. It is hoped that a search through this first online library will display as many articles as possible with the keyword multimodal semiotic reasoning on the subject of math or mathematics without a year limit. In addition, the search for reputable international journals is also through ERIC (The Education Resources Information Center). ERIC is an online digital library for educational research and information. This is consistent with the research focus on mathematics education. Therefore, a search for relevant manuscripts was carried out using the keyword "multimodal semiotic reasoning" on the subject of mathematics through Scopus, Science Direct, and ERIC.

Sampling and Data Collection Tools

Articles were collected from three journal databases, namely Scopus, Science Direct, and the Education Resources Information Center (ERIC) using the keyword "multimodal semiotic reasoning" on the subject of math or mathematics. Based on search results until April 2023, 32 articles were obtained from three databases. Furthermore, 32 articles were identified based on a) compliance with the criteria in Table 1; b) the presence of the same article appearing in the three databases.

Table 1: The criteria for inclusion and exclusion

Inclusion	Exclusion
Reasoning semiotic multimodal in math or mathematics	Reasoning semiotic multimodal outside math or mathematics
The research article	Other than research articles
The research design, findings, and conclusions are clearly stated (explicitly)	Research design, findings, and conclusions are not stated clearly (not explicitly)

The final search for manuscripts relating to reasoning semiotic multimodal with the subject of mathematics or mathematics education was obtained in 24 articles. A total of

24 articles were submitted 3 from Scopus, 9 from ERIC, and 12 from Science Direct.

Data analysis

A total of 24 articles were reviewed and data analysis was

performed following Tesch's (1990) (Yuliono et al., 2018). These stages are according to diagram 1 below.

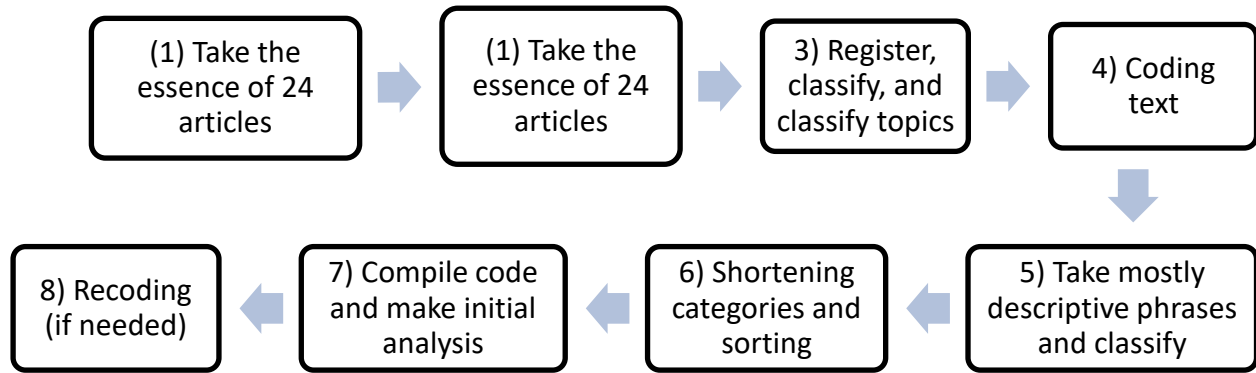


Diagram 1: Eight Stages of Data Analysis According to Tesch

Each manuscript was analyzed with a focus on a) kinds of mathematical reasoning involving symbols, and signs using multimodal, b) multimodal used in research, c) the most researched mathematical concepts and the mathematization processes that occur, and d) the role of semiotic reasoning multimodal in mathematics education. Kinds of mathematical reasoning involving symbols and signs using multimodal have two categories, namely Imitative Reasoning (IR) and Creative Mathematically Founding Reasoning (CMR). IR is also known as Memorized Reasoning (MR) and Algorithmic Reasoning (AR). IR is student reasoning that is often appropriate for routine tasks. The choice of MR strategy is based on memory, memory, answers, and strategy implementation through writing results. The choice of AR strategy is to memorize an algorithm, which is a sequence of rules for solving a certain

type of task (Mumu & Tanujaya, 2019). Whereas CMR is reasoning that is a novelty, makes sense, and has a mathematical basis. CMR is of two kinds, Local Creative Reasoning (LCR) and Global Creative Reasoning (GCR). LCR is a type of reasoning used to solve math problems; most of the solution uses formulas/algorithms that are commonly encountered by students, but there is an unknown solution step. On the other hand, in GCR-type reasoning, students need to think hard when solving math problems because all the necessary steps are unfamiliar to them. Mathematical tasks that can be solved using GCR are completely new tasks for students (Duijzer, Van den Heuvel-Panhuizen, Veldhuis, & Doorman, 2019; Kusaeri et al., 2022). This includes basic mathematics, novelty, and common sense. Kinds of mathematical follow diagram 2.

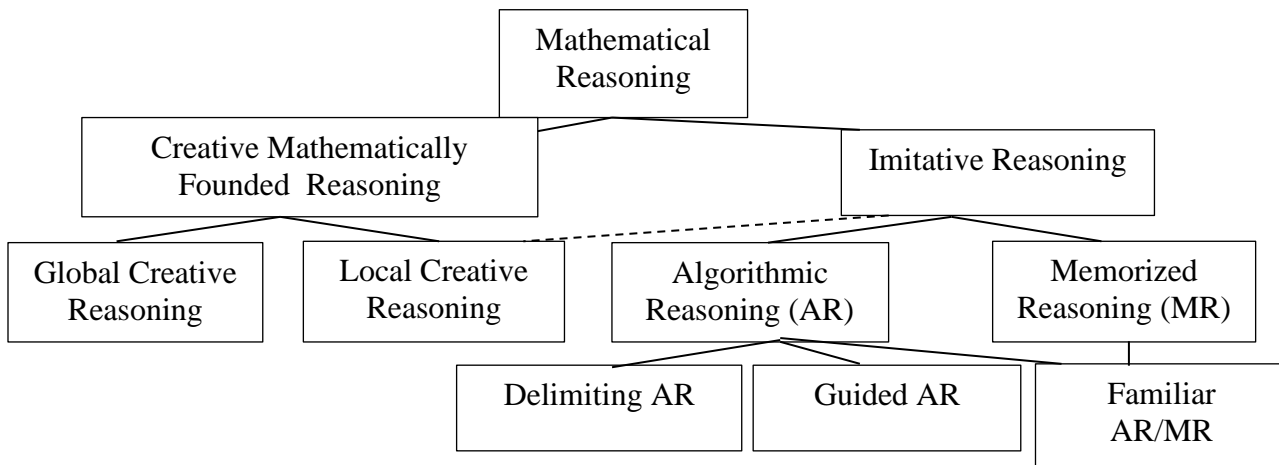


Diagram 2: Kinds of Mathematical Reasoning Involving Symbols, and Signs Using Multimodal (Kusaeri et al., 2022; Mumu & Tanujaya, 2019)

Based on the explanation regarding the reasoning above, this research will identify in depth the kinds of reasoning that involve mathematical signs or symbols using multimodal in each article. The resources used as part of multimodal will be identified based on the definition of multimodal. Multimodal is

the integration of various resources originating from written answer texts, oral answers, hand-arm-finger movements (gestures), facial expressions, images, artifacts, and concrete objects used by the subject (Arzarello et al., 2009; Danielsson & Selander, 2021). Mathematical concepts will be identified

based on existing concepts in mathematics, including numbers, algebra, geometry, measurement, and statistics (National Council of Teachers of Mathematics, 2000). While the mathematization process that occurs is identified based on horizontal and vertical mathematization. Activities that include horizontal mathematization are (1) Identifying special mathematics in a general context, (2) Schematics, (3) Formulation and visualization of problems in different ways, (4) Relation finding (relationship), (5) The discovery of regularity, (6) Introduction of isomorphic aspects in different problems, (7) Changing everyday problems into math problems, (8) Changing everyday problems into a known mathematical model. While activities that include vertical mathematization are, (1) expressing a relationship in a formula,

(2) proving regularity, (3) repairing and adjustment of the model, (4) the use of different models, (5) combining and integrating models, (6) formulation of a new mathematical concept, (7) generalization (Charlo, 2020). Analysis of the role of multimodal semiotic reasoning in mathematics education was identified based on findings, discussion results, article conclusions, and using NVivo version 12 pro software. In the NVivo application version 12 pro, data on word similarity frequency and cluster analysis can be obtained. Based on the cluster analysis, the role of multimodal semiotic reasoning in the field of mathematics education can be mapped. The steps for using the Nvivo application version 12 pro (Pratika et al., 2021) are as follows:

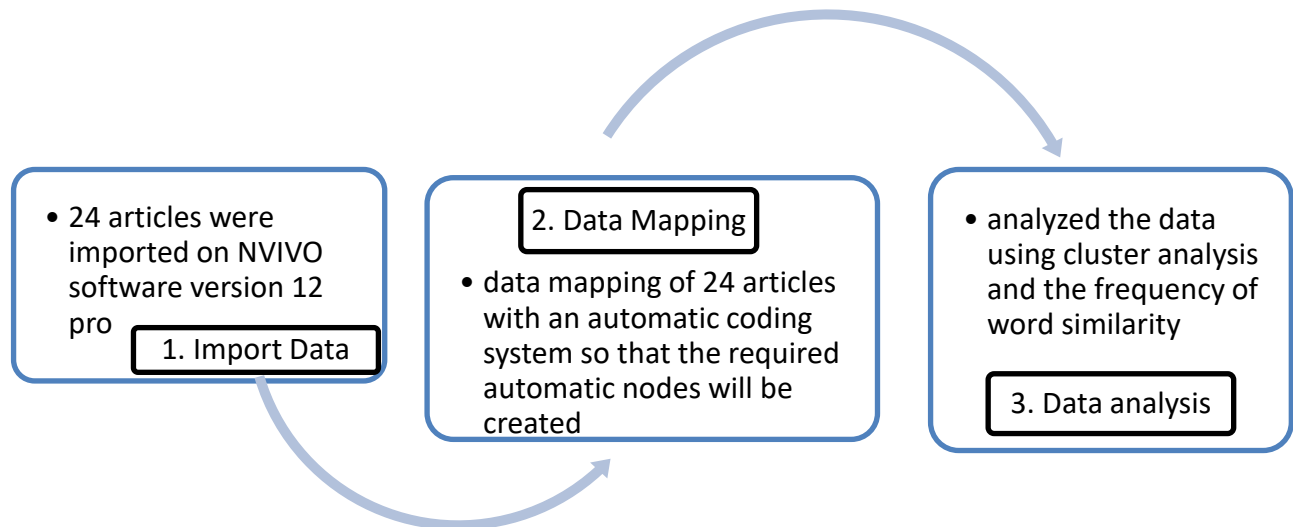


Diagram 3: Steps to use the NVivo version 12 pro application

Through the results of the similarity word image, data on the relationship between the 24 research articles were obtained. In addition, a hierarchical chart image will describe a pile of words in the form of thick or thin lines that connect interrelated words. The more piles of words from the 24 articles, the thicker the lines connecting the words will be. The research technique used to conclude is to identify the characteristics thoroughly and impartially in the text (Ratna et al., 2020).

FINDINGS

The results of the research on 24 articles explain a) kinds of mathematical reasoning involving symbols, and signs using multimodal, b) multimodal used in research, c) the most

researched mathematical concepts and the mathematization processes that occur, d) the role of semiotic reasoning multimodal in mathematics education. The four aspects are described using the subject of math or mathematics as part of that multimodal semiotic reasoning research conducted in mathematics education.

Kinds of Mathematical Reasoning Involving Symbols, and Signs Using Multimodal

This study analyzes the research design used in the 24 journals studied. Table 2 describes kinds of mathematical reasoning involving symbols, and signs using multimodal with mathematics as the subject.

Table 2: Kinds of Mathematical Reasoning Involving Symbols, and Signs Using Multimodal

Kinds of Mathematical Reasoning Involving Symbols, and Signs Using Multimodal	Number of Research	Percentage	Research Sample
Imitative Reasoning (IR)			
Algorithmic Reasoning (AR)	11	45,83%	(Oti & Crilly, 2021)
Memorized Reasoning (MR)	3	12,50%	(Nordin & Björklund Boistrup, 2018)
Creative Mathematically founded Reasoning (CMR)			
Global Creative Reasoning (GCR)	8	33,33%	(Lihua Xu et al., 2021)
Local Creative Reasoning (LCR)	2	8,33%	(Espeland et al., 2018)

Based on Table 2 above, it can be seen the types of semiotic reasoning using multimodal in mathematics education. The most dominant type of mathematical reasoning involving objects, symbols, and signs using multimodal is Algorithmic Reasoning (AR) (45.83%). A total of 11 articles use the AR reasoning type, there are 9 articles studied in the 2019-2022 period. The most researched topic is geometry. The application of the type of AR reasoning chosen by the researchers suggests that researchers focus on understanding an algorithm that states a sequence of rules for solving a particular type of task. Apart from geometry, the researcher uses mathematics in other

disciplines (18.18%), in the concept of numbers (9.09%), and in statistics (9.09%). In the next highest sequence, researchers used GCR (33.33%), MR (12.50%), and LCR (8.33%) reasoning. In the last three types of reasoning, researchers choose material or mathematical concepts that are more varied which will be explained in the next point.

Multimodal Used In Research

The research was conducted to analyze 24 multimodal-related articles used as illustrated in diagram 4 below.

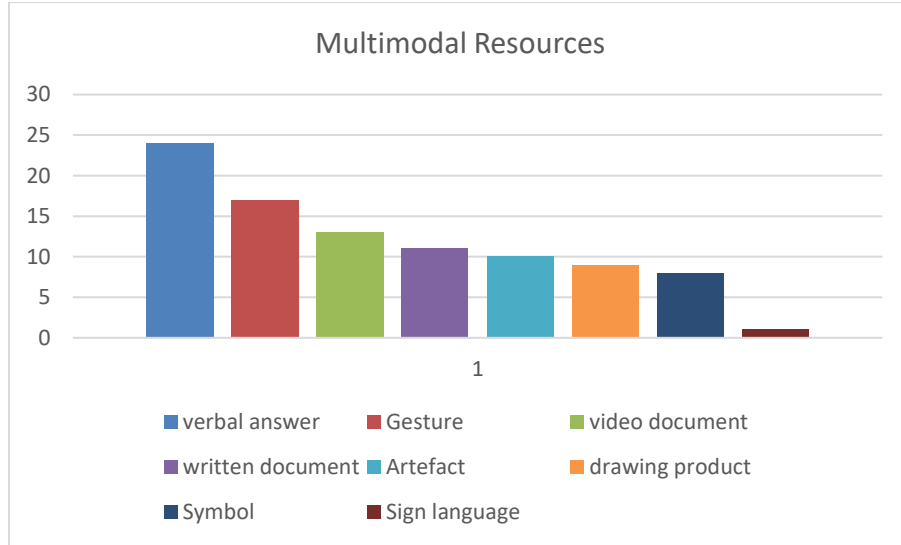


Diagram 4: Multimodal Used In Research

Based on diagram 4 it is known that the most widely used multimodal resource in multimodal semiotic reasoning research is oral answers. All articles (100%) have used sources of oral answers as part of multimodal semiotic reasoning. In addition, researchers use other multimodal such as gestures, video documents, written documents, artifacts, product images, symbols, and sign language. A total of 17 articles use gestures to explore multimodal semiotic reasoning. Researchers analyzed body movements, shoulders, eye gaze, and hand movements as part of the gesture. Apart from that, articles

published in the period 2020-2023 explore more multimodal from videos, written documents, artifacts, drawings, use of symbols, and sign language.

The Most Researched Mathematical Concepts and The Mathematization Processes That Occur

Based on the analysis of 24 articles it is known that mathematical concepts in multimodal semiotic reasoning research as shown in Table 3 below.

Table 3: Mathematical Concepts in Multimodal Semiotic Reasoning Research

Mathematical Concepts	Number of Research	Percentage	Research Sample
Mathematics Learning			
Geometry	10	41,67%	(Gürefe, 2022)
Algebra	2	8,33%	(Wilkie, 2019)
Math logic	2	8,33%	(Magnani, 2015)
math book	1	4,17%	(Daher & Abu Thabet, 2020)
Number	1	4,17%	(Chahine, 2013)
Application of Mathematics			
Field of Science	2	8,33%	(Lihua Xu et al., 2021)
Computer field	2	8,33%	(Ormerod et al., 2023)
English field	2	8,33%	(Moschkovich, 2015)
Field of Nursing	1	4,17%	(Björklund Boistrup & Gustafsson, 2014)
Field of Music	1	4,17%	(Espeland et al., 2018)

In general, as much as 63.64% of the 24 articles studied used mathematics learning materials in the classroom. Mathematical concepts include geometry, algebra, mathematical logic, and numbers. 36.36% of the 24 articles studied used the application of mathematics education in other fields. The application of mathematics to the fields of Science, Computing, English, Nursing, and Music is an important part of this multimodal semiotic research topic. In the future, research on multimodal semiotic reasoning with the subject of math or mathematics can use other mathematical materials,

such as measurement, and statistics. In addition, research on multimodal semiotic reasoning with the subject of math can be deepened and developed in other scientific fields. For example, Science material in elementary schools, and Mathematics in elementary schools.

Next, explain the mathematization process that occurs in the articles studied. Mathematization is identified based on horizontal, vertical, or vertical-horizontal mathematization criteria. The identification results are described in table 4 below.

Table 4: The Mathematization Process that Occurs

Mathematization	Number of Research	Percentage	Research Sample
Horizontal-Vertical	19	79,17%	(S Kjällander et al., 2021; Ubah & Bansilal, 2019)
Vertical	4	16,67%	(Moore-Russo & Viglietti, 2012; M Turgut, 2021)
Horizontal	1	4.17%	(Stein & Maier, 1995)

Based on table 4 it is known that 79,17% of researchers develop mathematics using vertical-horizontal mathematization. Researchers start by understanding the context that makes sense to students/participants. As done by one of the researchers. Mathematization begins with student-centered problem-solving in everyday contexts; utilizing inquiry and exploration to introduce concepts (Chahine, 2013). This context helps students to imagine and visualize abstract concepts. This is important for students, especially elementary school students. When students are in the process of taking action to reinvent mathematics it is called "mathematization". It should be noted that mathematization is identified based on criteria and also according to horizontal and vertical mathematization classifications. Horizontal mathematization is the process of students using their solutions to describe contextual problems with symbols. These symbols can be mathematical (formulas, algorithms, etc.) or not (numbers, diagrams, etc.). Students can present it explicitly, or even they can visualize or imagine it. In the world of symbols, students continue to use the mathematics they know to take action to discover new mathematical knowledge. This helps students answer problems that lie in the context of the problem. The process by which students take action in a mathematical system is called vertical mathematization.

The Role of multimodal semiotic reasoning in mathematics education

Determination of the role of multimodal semiotic reasoning in mathematics education was carried out using Nvivo 12 pro software. Nvivo facilitates queries that can be used to examine 24 articles related to trends, patterns, and meaningful relationships. The results of the auto code can be seen as hierarchical sequences, choices, or word repetitions. The results of this auto code can then be interpreted as the role of multimodal semiotic reasoning in mathematics education.

A total of 24 scripts were imported into the software. The next step is to obtain data mapping from 24 articles with the auto code system so that the required automatic nodes are made. Furthermore, the results of the frequency of word similarity and cluster analysis are obtained. Based on the results of the Nvivo analysis, a total of 934 words were obtained. Next, reduction of conjunctions, words that have the same basic word (for example students and students), and reduction of the similarity of words that are less than 200. In the end, a frequency diagram of the similarity of words is made which ultimately results in the 13 words below.

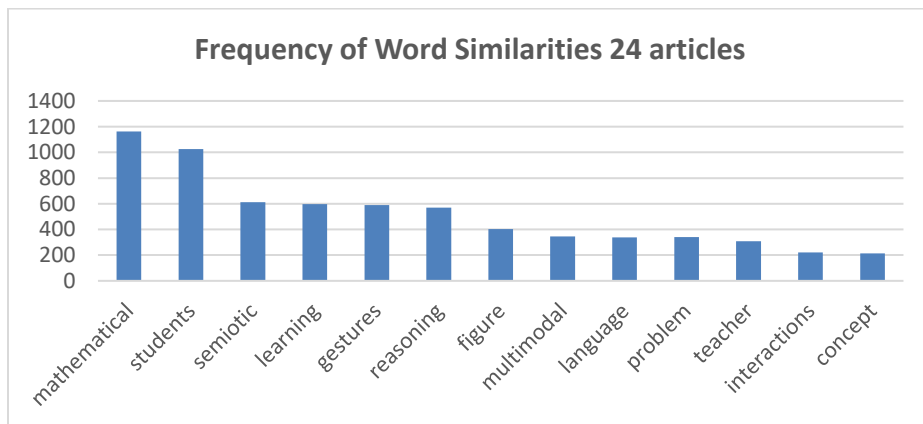


Diagram 5: Frequency of Word Similarities 24 articles

Based on the 13 words that appear in diagram 5, it can be seen that the word that appears most dominantly is "mathematical" and then the word "students". A total of 24 articles being researched discuss the topic of mathematics and

use students as research subjects. Then the 13 words that appear are followed by a cluster analysis query to find out trends, patterns, and meaningful relationships. The results of the cluster analysis are shown in Diagram 6 below.

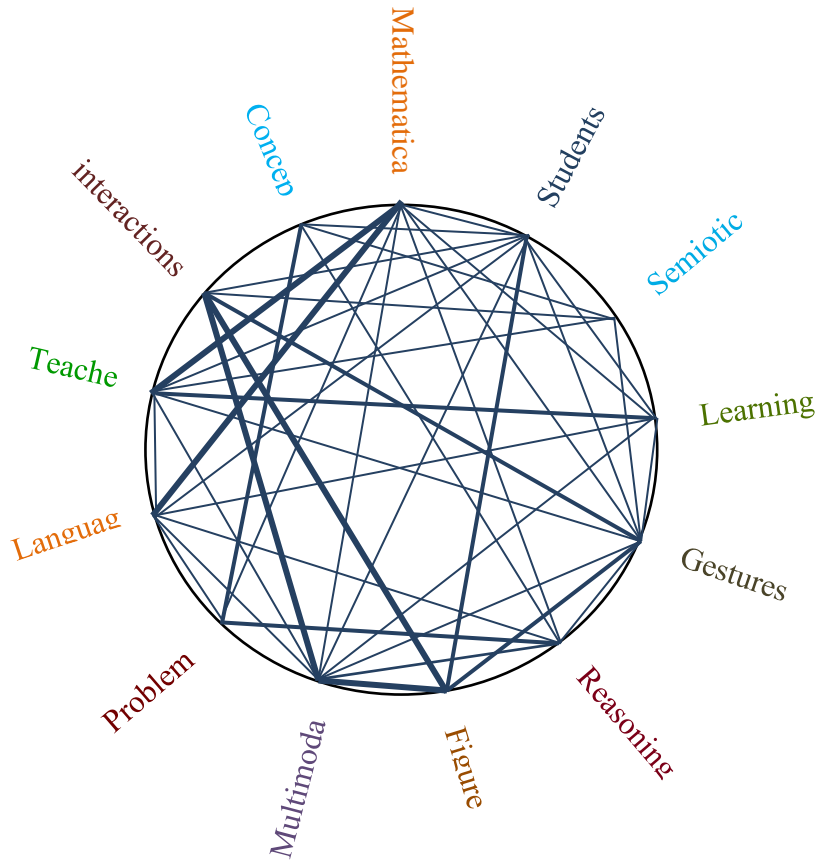


Diagram 6: Cluster analysis of 24 articles

The results of the cluster analysis in diagram 6 shows that there are thin lines and thick lines that connect interrelated words. There are 6 bold lines connecting related words. The thick line is a strong sign of the relationship between the two words. The 6 thick lines connect the words interactions-figure, interactions-multimodal, teacher-mathematical, language-

mathematical, problem-reasoning, and multimodal-figure. As for the thin lines, 40 lines connect words. The thin line indicates that there is a relationship between words but not very strong. Based on the similarity of words and bold lines, this directs the role of multimodal semiotic reasoning in learning mathematics as shown in Table 5 below.

Table 5: The Role of Multimodal Semiotic Reasoning in Mathematics Based on Cluster Analysis

Word	Related Words	Thick Lines	The Role of multimodal semiotic reasoning in Mathematics	Research Sample
Concept	students, semiotic, reasoning, problem	-	Knowledge	(Wilkie, 2019)
Language	teacher, mathematical, students, learning, reasoning, multimodal	language-mathematical		
Learning	gestures, multimodal, language, teacher, mathematical, students	-		
mathematical	teacher, language, problem, multimodal, reasoning, gestures, learning, students	mathematical-teacher mathematical-language		

Word	Related Words	Thick Lines	The Role of multimodal semiotic reasoning in Mathematics	Research Sample
Students	mathematical, concept, interactions, teacher, language, multimodal, figure, gestures, learning			
Teacher	mathematical, students, semiotic, learning, gestures, multimodal, language	teacher-mathematical teacher-learning		
Figure	multimodal, interactions, students, gestures	figure-interactions	Interactions	(C. L. Chen & Herbst, 2013)
Semiotic	concept, interactions, teacher, gestures reasoning, figure, multimodal, teacher, interactions, mathematical, students, semiotic, learning	-		
Gestures		-		
interactions	students, semiotic, gestures, figure, multimodal	interactions-figure interactions-multimodal multimodal-interactions		
multimodal	language, teacher, interactions, mathematical, students, learning, gestures, reasoning, figure	multimodal-figure	Math practices	(Björklund Boistrup & Gustafsson, 2014)
Problem	problem, concept, mathematical, reasoning	problem-reasoning		
reasoning	multimodal, problem, concept, mathematical, gestures	reasoning-problem		
Figure	multimodal, interactions, students, gestures	figure-multimodal		

In the end, after mapping the relationship between these words, there are 3 main roles of multimodal semiotic reasoning. The role in question is knowledge, interaction, and mathematical practice. In the future, the role of multimodal

semiotic reasoning may expand. This can be seen from the trend of increasing multimodal semiotic reasoning research with mathematics subjects in the year distribution as shown in diagram 7 below.

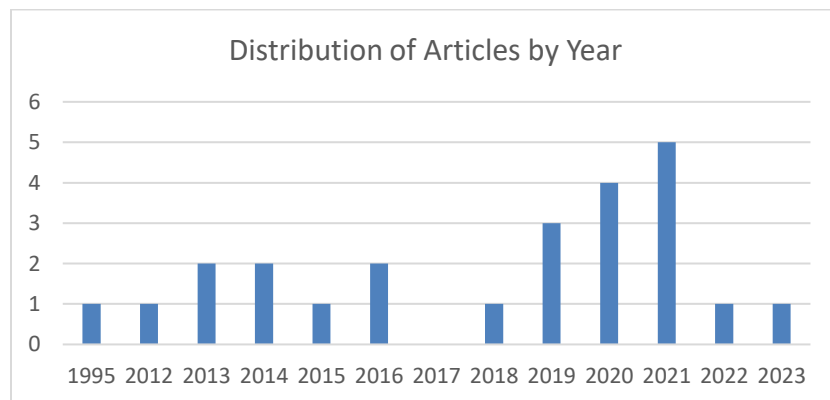


Diagram 7: Distribution of articles by year

Based on diagram 7 above, it is known that 45.83% of articles were published in 2020 – 2023. The development of learning that is carried out both online and offline and accommodates 21st-century skills, may be one of the supporters of the development of multimodal semiotic

reasoning with mathematics subjects. In general, the results of literature studies related to research on multimodal semiotic reasoning with the subject of mathematics provide positive energy in the development of the scientific field of mathematics. Not only in mathematics education in the

classroom but also outside the room, such as in the workplace.

DISCUSSION

Based on the analysis of 24 articles, there are 2 types of reasoning involving objects, symbols, and signs using multimodal. The types of reasoning are Imitative Reasoning (IR) and Creative Mathematically Found Reasoning (CMR). Imitative reasoning is reasoning based on previous experience and original effort. In general, imitative reasoning is only based on deep mathematical characteristics. Imitative reasoning consists of two categories, viz Memorized Reasoning (MR) and Algorithmic Reasoning (AR) (Mumu & Tanujaya, 2019; Sukirwan et al., 2018). Meanwhile, Creative Mathematically Found Reasoning (CMR) is differentiated according to Global Creative Reasoning (GCR) and Local Creative Reasoning (LCR).

In the IR type, 45.83% of the 24 articles studied used AR reasoning. The selection of AR strategies by researchers is used to memorize an algorithm which is a sequence of rules for completing a certain type of task. Examples are solving spatial literacy problems, computational tasks thinking and programming, graphic construction, and rectangular flat construction (Moore-Russo & Viglietti, 2012; Christine Wulandari Suryaningrum et al., 2020; Walkington et al., 2019; Wilkie, 2019). Strategy is carried out by carrying out each step that has been previously memorized. Researchers implement strategies by following a set of rules. Although the type of AR is part of imitative reasoning, in the case of solving non-routine problems, AR is very reliable (Sukirwan et al., 2018). As was done in the research on programming assignments of digital tools and polygon-non-polygon shapes for students with special needs (Gürefe, 2022; Susanne Kjällander et al., 2021). A total of 11 AR studies were conducted in 2020-2023. Apart from the AR type, the researchers also chose to use the MR type as much as 12.50% (Daher & Abu Thabet, 2020; Nordin & Björklund Boistrup, 2018).

Regarding the type of CMR, 33.33% of the researchers focused on GMR and 8.33% chose LMR. mathematics. CMR is reasoning based on creativity, the ability to produce original and meaningful work (Prain & Tytler, 2022; Sukirwan et al., 2018). There are four characteristics of CMR, namely 1) novelty, referring to new facts, namely the sequence of reasoning is created or recreated, 2) flexibility, meaning the ability to use different approaches and adaptations to certain problems, 3) plausibility, meaning that there are arguments that support the strategy selected and explain why the collection of conclusions is correct or reasonable, 4) a mathematical basis is built based on deep mathematical characteristics (Duijzer, Van den Heuvel-Panhuizen, Veldhuis, & Doorman, 2019; Ferguson, 2022). As research produces an eco-cognitive model, the Model for Analysing Digital Mathematics Teaching Material is based on a base layer, relational layer, process layer, and logical layer (Bergvall & Dyrvold, 2021; Magnani, 2015, 2016). The use of various types of reasoning involving signs, symbols, artifacts, and concrete objects as well as using multimodal (with various human resources) shows that multimodal semiotic reasoning is getting more and more attention in research in the field of mathematics education.

The reasoning strategy carried out in 24 studies prioritizes

the delivery of logical reasons to obtain conclusions. Obtaining conclusions cannot leave symbols or mathematical signs (Christine Wulandari Suryaningrum et al., 2020; Targon, 2019). Because symbols and signs in mathematics are part of the language of mathematics (Fatmanissa et al., 2019; Goldin, 2020; Salinas et al., 2016). The embodiment of reasoning involving signs and symbols is reflected in various forms of resources. Not only written answers, but in the form of oral answers (L Xu et al., 2021), pictures (Bobrova, 2021), videos (Melih Turgut, 2019), artifacts (Chahine, 2013) that students use, or even reflected through sign language as involvement. multimodal (Gürefe, 2022).

In multimodal use, the results of the analysis of 24 articles stated that oral answers were used by all articles. In addition to oral answers, the articles studied also explored a lot of multimodal from video documents, written documents, artifacts, product images, symbols, and sign language. The use of multimodal in multimodal semiotic reasoning research is very closely related to the field of work that will be involved. For non-teacher workers in multimodal research, semiotic reasoning is used to determine mathematical abilities related to the work they do (Björklund Boistrup & Gustafsson, 2014). Meanwhile, to clarify and focus attention on students with special needs, multimodal is manifested in the form of sign language (Gürefe, 2022). Although educational facilities for children with special needs have increased worldwide, there are concerns about providing children with special needs while studying. The results of the study show that teachers in classes with a high percentage of students with special needs tend to have the fewest qualifications and the needs of children with teacher needs are the greatest (Siregar et al., 2020). The need for teacher professional development has implications for education policies for children with special needs (Rettob et al., 2021). Based on these facts, it is estimated that future research involving multimodal students with special needs in classroom learning will increase. Apart from that, the use of multimodal for subjects or participants can be further developed with consideration of 21st-century developments (Duijzer & et al, 2019; Santos & Ortenzi, 2021).

In the 21st-century learning system, learning is no longer limited by space and time. The various materials selected by the researchers are part of the context used by the subject or participant. The hope, in addition to understanding and mastering the mathematical concepts they learn, is also directed so that the processes carried out by students are more meaningful. Participants or subjects not only memorize but the context of the material becomes an important part that drives them in carrying out the reasoning process. Until April 2023 it was discovered that mathematical concepts were still limited to some mathematical materials. As much as 41.67% is dominated by multimodal semiotic reasoning for geometric concepts (Gürefe, 2022). Furthermore, it also discusses the concept of algebra (Wilkie, 2019), mathematical logic (Magnani, 2015), mathematical practice in science (Lihua Xu et al., 2021), computers (Ormerod et al., 2023), and English (Moschkovich, 2015) each of 8.33%. In addition, it also discusses math books (Daher & Abu Thabet, 2020), number concepts (Chahine, 2013), math practice for nurses (Björklund Boistrup & Gustafsson, 2014), and music (Espeland et al., 2018) each of 4.67%.

The materials used in research on multimodal semiotic reasoning have gone through a series of mathematization processes. As much as 75% carry out the horizontal-vertical mathematization process. As much as 16.67% did vertical mathematization and 4.17% chose to do horizontal mathematization. Several different possibilities in the mathematization process can occur. Horizontal and vertical processes in complex and complicated paths can be chosen by someone (Charlo, 2020). However, another possibility is to take the horizontal process shortly and simply, even though the vertical process is taken through complicated paths or vice versa (Tunç-Pekkan, 2015). In general, learning mathematics tends to use ways that students can easily understand (Duval, 2017). For most of the exercises, one prefers to go short and the method is a horizontal section followed by some vertical sections or vice versa. Some of the possibilities pursued by students and other circles are what make it possible to know the process of mathematization that takes place in multimodal semiotic reasoning research.

The role of multimodal semiotic research becomes very important when seen in the results of table 5. The results of the analysis of 24 articles state that the role of multimodal semiotic reasoning in mathematics education is for knowledge, interaction, and practice of mathematics. The role of multimodal semiotic reasoning according to Wilkie (2019) is to facilitate students in reasoning on functional material (linear and nonlinear) with different contexts. This article focuses on learning contexts and involves different types of assignments and presentations to visually explore functional relationships. In this case, semiotics is accommodated through the use of various contexts in learning. Meanwhile, multimodal is manifested through the visual aspects of various assignments and presentations. The research findings support the assertion that a reluctance to emphasize the visual aspects of learning mathematics is a serious barrier to student learning.

In its role in interaction, one of the research samples (C.-L. Chen & Herbst, 2013) conducted research to identify forms of interaction in diagrammatic material. Specifically, how do students display their thoughts openly through the use of multimodal representations? The results of the study explain interactions in gestural and verbal forms, and how multimodal interactions with diagrams can reveal students' reasoning. The use of signs on objects used to make diagrams becomes the semiotic element discussed. A further finding in this study is that the use of gestures can contribute to understanding students' thinking and learning in the classroom (Swidan et al., 2022). Furthermore, the role of multimodal semiotic reasoning in mathematics education is in the form of mathematical practice. The data consist of videos and transcribed interviews from the work of two lorry loaders, and a nurses' aide at an orthopedic department. In the analysis, we adopt a multimodal approach where all forms of communicative resources (e.g., body, speech, tools, symbols) are taken into account (Lim & Toh, 2020). We also incorporate the institutional norms of workplace activities into the analysis. We coordinate a multimodal social-semiotic perspective with a learning design sequence model (Danielsson & Selander, 2021) which makes explicit the institutional framing. Adopting this framework enables us to understand learning as communication within a domain, with an emphasis on content matters, interpersonal

aspects, and roles of communicative resources and artifacts. We describe tentative themes and explain how workplace-specific resources for measuring deliver the efficiency and function of workplace mathematics practice. In this study, the analysis was carried out by adopting a multimodal approach that takes into account all forms of communication resources (eg, body, speech, tools, symbols) (Holsanova, 2020; Pacheco et al., 2021).

Research into the practice of mathematics in the workplace is part of the development of research methods and analytical frameworks about the complexity and power of mathematical practice beyond school mathematics (Font et al., 2013). The complexity of workplace mathematics can be brought into school mathematics on the condition that it accommodates the knowledge of workers in various sectors. There are two different perspectives on math in school versus math in the workplace. Math at work uses more advanced basic math than in the classroom (Cruz, 2018). Work-related mathematics is data-rich, predictive, technology-dependent, and bound to useful applications (Elkjær & Jankvist, 2021). Work contexts often require multi-step solutions to open problems, a high degree of accuracy, and precise attention to required tolerances. Some of these things are not found in regular classroom exercises. Given the complexity of mathematics in the workplace, the use of a multimodal approach is essential for understanding communication within the domain, with an emphasis on content issues, interpersonal aspects, and the role of communicative sources and art facts (Al-Mutawah et al., 2022; Björklund Boistrup & Gustafsson, 2014; Wood, 2012). These facts of necessity then become a stimulus for further research in the future and further enhance the role of multimodal semiotic research in mathematics. This is reinforced by research data based on the distribution of research years. A total of 11 studies of 24 studies were conducted in 2020-2023.

CONCLUSION

Multimodal semiotic reasoning with mathematics subjects is interpreted as a decision-making process related to objects, symbols, or signs through integrating various sources originating from student writing, oral answers, movements such as hand-finger-arm (gestures), student facial expressions, expressions, pictures, concrete objects, artifacts used by students have a positive impact on scientific development in the field of mathematics. Based on the results of a literature study on 24 manuscripts from several Scopus, Science Direct, and ERIC databases, multimodal semiotic reasoning research also shows a major influence in the field of mathematics education. The results of the literature study also note that a) the most dominant type of mathematical reasoning involving objects, symbols, and signs using multimodal was Algorithmic Reasoning (AR), b) the most multimodal source was verbal answers, c) the concept of geometry through a process of vertical-horizontal mathematization was the most researched, and 4) the biggest role of multimodal semiotic reasoning for the benefit of knowledge of mathematics education. This is reinforced by research data based on the distribution of research years. A total of 11 studies of 24 studies were conducted in 2020-2023.

SUGGESTION

Mathematics learning that is currently carried out is not only focused in the classroom but has also utilized virtual space. Learning has made maximum use of all the resources that students have. Therefore, students' abilities are not only measured in writing but in oral answers, movements such as hand-finger-arm (gestures), student facial expressions, pictures, concrete objects, and artifacts used by students have a positive impact on the development of science in mathematics (multimodal). This development provides interesting facts related to students' semiotic reasoning abilities in learning mathematics. Taking these developments into account, further research can be developed with other types of reasoning, other multimodal resources (eg other sign languages), and different mathematical materials. Thus, the role of multimodal semiotic reasoning can develop. The hope is that it can be used to develop mathematical knowledge, both inside and outside the classroom (such as in the world of work). In addition, multimodal semiotic reasoning provides an overview of the needs in learning and applying the field of mathematics. Needs relating to the development of learning media, learning methods/models, learning assessment, or other components in mathematical activities.

LIMITATION

This article has limitations in the form of limited manuscript analysis until April 2023, limited access to only three databases, and limited articles in English. This provides space for further research by involving databases of other journals and journals that use articles other than English. Thus, information about multimodal semiotic reasoning can be known more deeply.

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