

# The controversial mathematical reasoning process of prospective teachers when solving mathematical problems

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**ABSTRACT:** An important aspect of the learning process is reasoning. One of the studied reasonings is controversial reasoning. Controversial reasoning can be used to determine the understanding of prospective teachers. The purpose of this study is to explain the controversial reasoning of prospective teachers when solving problems. This research is descriptive qualitative. The instruments used are tests and interviews. There are 50 prospective teachers who are given a problem, then their work is analyzed to find out their reasoning. Data analysis in this study is a) reviewing tests and interviews, b) determining the data used, c) analyzing test and interview results, d) making patterns, and e) presenting data. The conclusion of this study is the controversial reasoning process of prospective teachers, namely a) logically concluding, b) explaining the model, facts, and concept relationships, c) determining the conjecture, and d) using the conjecture to analyze the situation and make an analogy with the given problem. Suggestions for other researchers are to develop appropriate instruments for generating controversial reasoning for prospective teachers. In addition, after prospective teachers solve controversial problems, they can explore their critical and creative thinking processes.

*Keywords:* Mathematical reasoning, prospective teachers, problem

## 1 INTRODUCTION

Reasoning is one of the goals of the curriculum and an important element in mathematics education (Brumbaugh *et al.* 2020; Herbert and Brown 2020; Martin and Kasmer 2020; Rott 2020). Reasoning is used in solving mathematical problems so that it can facilitate solving (Bozkuş and Ayvaz 2018; Mariotti 2019; Zhou *et al.* 2018). Because of the importance of reasoning in mathematics education, one of the main foundations of prospective teachers must be emphasized (Susiswo *et al.* 2021). Prospective teachers are one of the determinants of the success of the learning process (Fernández *et al.* 2020; Nelson and Hawk 2020; Simon 2020).

The mathematical reasoning of prospective teachers is still relatively low (McCrorry and Stylianides 2014; Hohensee 2017). Based on research results (Jeannotte and Kieran 2017) state that prospective teachers have not had the opportunity to learn about how to reason and prove. Research results (Battista 2017) mentioned that prospective mathematics teachers are still weak in reasoning but they still have confidence in the learning process carried out.

Various attempts have been made to correct the weak reasoning of prospective teachers. Many studies have been carried out to improve the reasoning of prospective teachers. Among other things (Karatoprak *et al.* 2015; Kertil *et al.* 2019; Mulenga and Marbán 2020; Thanheiser *et al.* 2016), research (Karatoprak *et al.* 2015) mentioned that training had been carried out for prospective teachers to develop reasoning. Researchers (Thanheiser *et al.*

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2016) conducted studies on reasoning strategies as the basis of knowledge of prospective teachers. Researchers (Kertili *et al.* 2019) describe the development of covariational reasoning through three categories, namely the identification of variables, how to coordinate variables, and measuring the level of change. However, no one has used controversial reasoning to strengthen the mathematical reasoning of prospective teachers.

Controversial reasoning causes differences of opinion in which there is a process of arguing (Goldberg and Savenije 2018; Mueller and Yankelewitz 2014a; Simic-Muller *et al.* 2015). In controversial reasoning, some arguments give rise to different ideas. This causes a person to be more leveraged in the process of solving a given problem.

Controversial reasoning research has been carried out by Maria Lim (2013), and the result is that the process of controversial reasoning can help someone's thought process. Mueller and Yankelewitz (2014b) explained that students need controversial reasoning so that they are more critical and can explain it to others. Research results (Mueller and Yankelewitz 2014b) mentioned that students' controversial reasoning tends to be below, and further research is needed. Prospective teachers are one of the important aspects of maximizing students' reasoning. This study is a follow-up to research on controversial reasoning but on aspiring mathematics teachers. This is done because no research looks at the controversial reasoning process of prospective mathematics teachers. The urgency of this research is that research on controversial reasoning has never been done even though the results of previous research with controversial reasoning can make prospective teachers more critical. Based on the above background, this study aims to describe the controversial reasoning process of prospective teachers in solving math problems.

## 2 METHOD

This research is descriptive with a qualitative approach. The research instruments used were tests and interviews. The test is used to find out the process of working with prospective teachers in solving the problems given and seeing the controversial reasoning process. The test consists of one controversial issue that the prospective teacher must resolve. The test was completed within 20 minutes, after which it was seen how his reasoning used the appropriate criteria. Interviews are used to deepen information about the reasoning of prospective teachers.

**Identitas Diri**

NAMA :  
 NIM :

**Selesaikan setiap permasalahan yang diberikan di bawah ini pada lembar jawaban yang sudah tersedia!**

**Masalah 1**

Ketika guru memberikan soal kepada siswa, sederhanakan bentuk aljabar  $\frac{2x^3-3xy-2y^3}{x-2y}$ . Siswa menyelesaikannya dengan memfaktorkan pembilang dan membagi bentuk yang sama dengan penyebutnya.

$$\frac{2x^3-3xy-2y^3}{x-2y} = \frac{2x^2(2x+y) - 2y^3}{(x-2y)} = 2x + y$$

Ada siswa lain yang bertanya "apa boleh  $(x-2y)$  dibagi dengan  $(x-2y)$ ? Bagaimana kalau  $x = 2y$ ?"

- Menurut Anda, apakah pertanyaan siswa tersebut masuk akal? Jelaskan!
- Seandainya Anda sebagai guru dari siswa tersebut, apa yang dapat Anda jelaskan terkait dengan masalah tersebut, supaya siswa dapat memahaminya secara baik?

Figure 1. Problems given.

This study involved 50 prospective teachers who had been given problems, then the results of the work of prospective teachers were corrected to find out the controversial reasoning experienced by prospective teachers. The criteria for prospective teachers to experience controversy are the occurrence of cognitive conflicts and arguing during the interview process.

a copy of the problem given:

Problem 1: The form of algebra (implicitly controversial)

When the teacher gives questions to students, simplify the algebraic form  $\frac{2x^2 - 3xy - 2y^2}{x - 2y}$ . Students solve it by factoring in the numerator and dividing the same shape as the denominator.

$$\frac{2x^2 - 3xy - 2y^2}{x - 2y} = \frac{(x - 2y)(2x + y)}{(x - 2y)} = 2x + y$$

Other students ask, "Can (x-2y) be divided by (x-2y)? How about x = 2y?"

### 3 RESULTS AND DISCUSSION

Based on the results of the study, there was one prospective teacher who experienced controversy, so it could be seen how the controversial reasoning process was. The following are the results of the teacher candidates' answers.

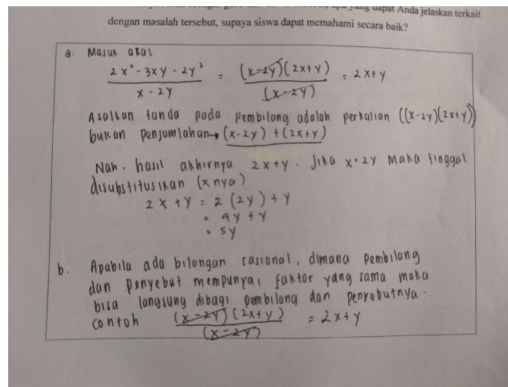


Figure 2. Teacher candidate answers.

From the answers of the prospective teachers, each indicator of controversial reasoning can be seen. The following is an explanation:

#### 3.1 Experiencing controversy when solving a given problem

This process can be seen during interviews, researchers conduct interviews after prospective teachers work on a given problem. From the results of the interviews, it can be seen that prospective teachers experience controversy. In addition, prospective teachers had time to ask researchers during interviews, the following is an excerpt:

C: Is the question correct, ma'am?

P: Yes, because it is appropriate. Why do you ask that?

C: I think there are additional conditions.

From the snippet of the dialogue, it can be seen that there is a cognitive conflict because prospective teachers ask for additional requirements to solve the problem.

The process of cognitive conflict occurs when a person experiences a conflict within himself (Devine *et al.* 2018). In cognitive conflict, it allows someone to convey their ideas by arguing with others (Bregant 2014). In line with research results (Simic-Muller *et al.* 2015) which state that prospective teachers should be given problems that trigger controversial reasoning (Kuhn 2010).

The problems given can give rise to various arguments, and controversial reasoning that affect the maximum learning process (Aksu *et al.* 2016; Miller and Flores 2012; Oulton *et al.* 2004; Simic-Muller *et al.* 2015). Indicators of prospective teachers are controversial, namely experiencing cognitive conflicts, differences of opinion, and trying to solve the problem (Kello 2016).

### 3.2 Concluding logically

When solving a given problem, prospective teachers draw logical conclusions. The conclusions of prospective teachers can be seen in the answers in Figure 3 as follows.

Asalkan tanda pada pembilang adalah perkalian  $((x-2y)(2x+y))$   
bukan penjumlahan  $\rightarrow (x-2y) + (2x+y)$

Figure 3. Logical conclusion.

Prospective teachers conclude that the operation can be used for multiplication and not addition. Furthermore, researchers deepen the information through interviews as follows.

Q: Why did you write the sign as multiplication?

C: Because addition cannot be used.

Q: Why?

C: Later, the value will not be the same. Making logical conclusions is one alternative that can be used in solving problems (Simic-Muller *et al.* 2015). Concluding is one of the processes in solving a given problem (Rosyadi *et al.* 2021; Rosyadi and Sa'dijah *et al.* 2022). This is in accordance with the results of the study (Faizah 2009), which states that in deductive and factual reasoning, there is a process of concluding solving the given problem (Rosyadi and Sa'dijah *et al.* 2022). The process of concluding a fact hereinafter referred to as reasoning also occurs in research (Basri *et al.* 2019). Research also mentions that abductive reasoning is also used to solve problems in which conclusions are drawn.

### 3.3 Explaining the model, facts, or relationships between concepts.

From the results of the answers of prospective teachers can be seen models, facts, and the relationship between concepts. Look at Figure 4 below, prospective teachers explain the link between rational numbers, factors, and division of numbers.

b. Apabila ada bilangan rasional, dimana pembilang dan penyebut mempunyai faktor yang sama maka bisa langsung dibagi pembilang dan penyebutnya.  
contoh  $\frac{(x-2y)(2x+y)}{(x-2y)} = 2x+y$

Figure 4. Explaining the relationship between concepts.

Linking between concepts is one alternative to understanding prospective teachers in solving problems (Baroody and Bartels 2020). The facts and models that exist in the given problem can be presented in a relationship between concepts (Blomhøj and Jensen 2003; Vinner and Dreyfus 2020).

An explanation of the interrelationships between concepts is an important part of solving problems (Baroody and Bartels 2020; Hong and Choi 2011; Vanscoy 2019). According to (Baroody and Bartels 2020) The relationship between the two concepts is important in completing the given task (Rosyadi and Sa'dijah *et al.* 2022). By understanding the relationship between concepts, you can solve problems and understand the ideas given (Vanscoy 2019).

### 3.4 *Making conjectures and evidence*

Allegations and evidence of prospective teachers can be seen from the results of interviews with researchers. The results of interviews with researchers resulted in the following conversations.

Q: What temporary conclusions can you get after working on the problem?

C: Division by the same number can only be done for multiplication operations.

P: okay, then there are others?

C: Oh yes, the number cannot be 0

From the conversation, the prospective teacher can mention two assumptions from the process of solving the given problem. This is in accordance with research (Hariyani *et al.* 2016) which states that the process of making assumptions is one part of the process of solving problems. The process of making guesses is one of the unique phases in the thinking process (Hariyani *et al.* 2016). In making assumptions, the solution strategy chosen will be different from the justification stage (Hidayah *et al.* 2020).

### 3.5 *Use relationship patterns to analyze situations, make analogies, or generalize.*

The pattern of relationships used by prospective teachers in conducting the generalization process can be seen in Figure 4. *The conversation between researchers and prospective teachers* can be seen as follows.

Q: From the conclusions that have been made, mentioning that division by the same number can only be done for multiplication operations, how do you solve other equivalent problems?

C: I tried to use the material that I got first and relate it to the question

Q: Then what?

C: Suppose I can't finish, I try to use trial and error

The process of making generalizations made by prospective teachers is used to make it easier to understand the problem and be able to solve other equivalent problems (Fensham and Bellocchi 2013; Rittle-Johnson and Schneider 2014). In the generalization process, there is a process of learning, understanding, and manipulating (Saxton *et al.* 2019).

The findings in this study are the stages of reasoning that have been described by Rosyadi *et al.* (2022) and can be used in other relevant studies. In making conjectures and evidence, prospective teachers have not found the right pattern (Ulger 2018). This is because the concept of prerequisites and linkage of the material is not maximized. For further research, appropriate learning methods can be developed for making conjectures and evidence (Martin and Kasmer 2020; Susiswo *et al.* 2021).

The limitation of this research is that it only uses controversial subjects, then it can also be developed for other subjects. Other research can also look at gender and the learning styles of prospective teachers.

## 4 CONCLUSIONS AND SUGGESTIONS

Based on the results of the study, it can be concluded that the controversial reasoning processes of prospective teachers include: a) there is a controversial process when solving a given problem, b) draws conclusions logically, c) provides explanations about models, facts, or relationships between concepts, d) make conjectures and evidence, and e) use relationship patterns to analyze situations, make analogies, or generalize. Suggestions for other researchers are to develop appropriate instruments in bringing up the controversial reasoning of prospective teachers. In addition, after prospective teachers solve controversial problems, they can explore their critical and creative thinking processes.

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