Al-Bidayah : jurnal pendidikan dasar Islam
Volume 15, Number 2, December 2023
ISSN: 2549-3388 (online)
Available online at https://jurnal.albidayah.id
Submitted: October 23, 2023, Accepted: December 30, 2023

PATTERN OF SEMIOTIC REASONING IN SOLVING TWO-DIMENSIONAL SHAPE PROBLEMS: A MULTIMODAL ANALYSIS

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DOI: 10.14421/al-bidayah.v15i2.1049

ABSTRACT

Semiotic reasoning is the process of drawing a conclusion based on objects, signs, and students' interpretations of the meaning of signs. This process is reflected through written, verbal, and gestural answers (multimodal). Therefore, this research aims to describe the patterns of students' semiotic reasoning in solving two-dimensional shape problems based on a multimodal analysis. This study was conducted qualitatively using an exploratory descriptive approach. The research was conducted by involving 2 participants from the third grade of Muhammadiyah Elementary School, Malang. The instruments used were observations, interview guidelines, and written exam questions. Observations were made when the participants solved problems in writing and during interviews. The interviews were conducted to confirm the written answers. The written test consisted of one question about the perimeter of a two-dimensional shape. The research results of the two third-grade students were analyzed using four stages of (1) data transcription, (2) sorting and selecting appropriate data, (3) data presentation, and (4) drawing conclusions. The research obtained two patterns of semiotic reasoning used by the participants. The first pattern was analysis, analysis-generalization, integration, and justification. The second pattern was analysis, analysis-generalization, integration, analysis, integration, integration, and justification. In addition, it was found that the first pattern gave the wrong final answer. In the first pattern, especially at the stage of implementing the problem-solving plan, the participant made a mistake in the integration indication, resulting in a wrong answer. Follow-up research needs to be carried out to determine the quality of students' reasoning ability when solving multimodal-based mathematical problems.

Keywords: multimodal; problem-solving; semiotic reasoning; two-dimensional shapes

INTRODUCTION

Reasoning is the process of drawing conclusions based on facts of which the truth has been previously proven or assumed. While doing reasoning, students have a chance to explore their ability or experience in understanding the concept of mathematics. To date, the reasoning ability of Indonesian students can be identified in the *Trend in International Mathematics and Science Study* (TIMSS) test which was last carried out 4 years ago[1]. The last time Indonesian students took the TIMSS test was in 2015. Meanwhile, in 2019, Indonesia did not join the test. The next TIMSS test will be conducted in 2023. The TIMSS test results in 2015 showed that the capability of

[1] Michael O Martin and Matthias Von Davier, *TIMSS 2023 Assessment Frameworks TIMSS 2023 Assessment Frameworks*, 2022; Ina V.S Mullis et al., "Timss 2015 International Results in Mathematics," *IEA: TIMSS & PIRLS International Study Center*, 2015, 1–971.

reasoning of Indonesian students ranked 47th among 50 countries. This shows that the ability to reason remains low. A country's ranking in this TIMSS test can be used as an evaluation material for the policy to improve the quality of mathematics learning in the future[2].

To prepare for the TIMSS 2023 test, TIMSS has established the TIMSS 2023 framework. One part of the TIMSS 2023 framework is the reasoning framework that includes analysis, generalization, integration, and justification. TIMSS does not explain the sequence of this reasoning framework but ensures that these components are present as the indicators of reasoning. Several researchers explain that students' reasoning ability are used to analyze problems, solve problems, draw conclusions, and express ideas logically[3]. When solving mathematical problems, students need objects, symbols, and signs. This is because the use of objects, signs, or symbols in learning mathematics in elementary school is needed at the development stage of elementary school students, which is concrete operations[4]. The use of objects, signs, or symbols is called semiotics[5]. Semiotics is a science that studies the production of signs and symbols to communicate information. Reasoning that involves objects, symbols, or signs is called semiotic reasoning[6].

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[2] George E. Deboer, "The Globalization of Science Education," *Journal of Research in Science Teaching* 48, no. 6 (2011): 567–91, https://doi.org/10.1002/tea.20421; Rachel Louise Geesa et al., "Exploring Factors of Home Resources and Attitudes towards Mathematics in Mathematics Achievement in South Korea, Turkey, and the United States," *Eurasia Journal of Mathematics, Science and Technology Education* 15, no. 9 (2019), https://doi.org/10.29333/ejmste/108487.

[3] Thomas Hunt et al., "Trends in International Mathematics and Science Study (TIMSS)," *Encyclopedia of Educational Reform and Dissent*, 2013, 562–69, https://doi.org/10.4135/9781412957403.n438; Martin and Davier, *TIMSS 2023 Assessment Frameworks TIMSS 2023 Assessment Frameworks*.

[4] M. Fadiana et al., "How Concrete Operational Student Generalize the Pattern?: Use Semiotic Perspective," *Journal of Physics: Conference Series* 1188 (2019): 1–8, https://doi.org/10.1088/1742-6596/1188/1/012032.

[5] Silvia Klettner, "Form Follows Content: An Empirical Study on Symbol-Content (in)Congruences in Thematic Maps," *ISPRS International Journal of Geo-Information* 9, no. 12 (2020), https://doi.org/10.3390/ijgi9120719; Yulia Erokhina, "Semiotic Interpretation of the Sign 'Ecclesiastical Court' Within the Framework of Legal Precepts in Terms of Temporality and Spatiality (Case of Russia)," *International Journal for the Semiotics of Law* 34, no. 3 (2021): 783–802, https://doi.org/10.1007/s11196-020-09756-z.

[6] C. W. Suryaningrum and Y. D.W.K. Ningtyas, "Multiple Representations in Semiotic Reasoning," *Journal of Physics: Conference Series* 1315, no. 1 (2019), https://doi.org/10.1088/1742-6596/1315/1/012064; V Prain, J Ferguson, and P.-O. Wickman, "Addressing Methodological Challenges in Research on Aesthetic Dimensions to Classroom Science Inquiry," *International Journal of Science Education*, 2022, https://doi.org/10.1080/09500693.2022.2061743.

During the semiotic reasoning process, students display their understanding by integrating the various resources they have. The sources that students have, such as written answers, spoken answers, and gestural answers, are called multimodal[7] In-depth multimodal engagement learning shows understanding in learning, which accommodates the role of semiotics for assessing, supporting, and communicating learning in class. Multimodal places teacher assessment not only in a written form but also in other documentations, such as activity videos, recorded interviews, and movements made by students[8]. Students not only play an active role by integrating all sources of power they possess.

The search for research articles on Scopus, ERIC, and ScienceDirect until April 2023 with the keywords "multimodal semiotic reasoning" and the subject "mathematics" resulted in 24 articles (3 from Scopus, 9 from ERIC, and 12 from ScienceDirect). A literature study was then conducted on these 24 articles. The results showed that a) the most dominant type of mathematical reasoning involving objects, symbols, and signs using multimodal is the Algorithmic Reasoning (AR)[9], b) the most dominant source of multimodal is verbal answers[10], c) the most widely researched geometric concept is

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[7] Kristina Danielsson and Staffan Selander, *Multimodal Texts in Disciplinary Education*, *Multimodal Texts in Disciplinary Education*, 2021, https://doi.org/10.1007/978-3-030-63960-0; G Reershemius, "Lamppost Networks*: Stickers as a Genre in Urban Semiotic Landscapes," *Social Semiotics* 29, no. 5 (2019): 622–44, https://doi.org/10.1080/10350330.2018.1504652.

[8] E L Miller, "Negotiating Communicative Access in Practice: A Study of a Memoir Group for People With Aphasia," *Written Communication* 36, no. 2 (2019): 197–230, https://doi.org/10.1177/0741088318823210; Candia Morgan and Chronis Kynigos, "Digital Artefacts as Representations: Forging Connections between a Constructionist and a Social Semiotic Perspective," *Educational Studies in Mathematics* 85, no. 3 (2014): 357–79, https://doi.org/10.1007/s10649-013-9523-1; M Riaz, "Semiotics of Rape in Pakistan: What's Missing in the Digital Illustrations?," *Discourse and Communication* 15, no. 4 (2021): 433–57, https://doi.org/10.1177/17504813211002036.

[9] Alfred Oti and Nathan Crilly, "Immersive 3D Sketching Tools: Implications for Visual Thinking and Communication," *Computers and Graphics (Pergamon)* 94, no. April (2021): 111–23, https://doi.org/10.1016/j.cag.2020.10.007.

[10] C.-L. Chen and P Herbst, "The Interplay among Gestures, Discourse, and Diagrams in Students' Geometrical Reasoning," *Educational Studies in Mathematics* 83, no. 2 (2013): 285–307, https://doi.org/10.1007/s10649-012-9454-2; Magne Espeland, Kari Smith, and Øystein Kvinge, "Performing the Pre-Formed: Towards a Conceptual Framework for Understanding Teaching as Curricular Transformation," *Designs for Learning* 10, no. 1 (2018): 29–39, https://doi.org/10.16993/dfl.83; Nejla Gürefe, "How Must a Polygon Be According to Hard of Hearing Students? An Investigation with a Semiotic Approach," *Journal of Research in Mathematics Education* 11, no. 2 (June 24, 2022): 180–213, https://doi.org/10.17583/redimat.6097; Lihua Xu, Joseph Ferguson, and Russell Tytler, "Student Reasoning About the Lever Principle Through Multimodal Representations: A Socio-Semiotic Approach," *International Journal of Science and Mathematics Education* 19, no. 6 (2021): 1167–86, https://doi.org/10.1007/s10763-020-10102-9.

through the vertical-horizontal mathematization process[11], d) the role of multimodal semiotic reasoning is for mathematical knowledge, and e) qualitative research is the most widely used approach by researchers.

Multimodal semiotic research[12] has been done for elementary school students but it has not revealed the type of students' reasoning. Research on the multimodal use embedded to digital teaching materials suggested that students' reasoning abilities are used in mathematics learning[13]. Multimodal engagement is even able to reveal the mathematical semiotic reasoning ability of workers, such as nurses[14]. Research on multimodal semiotic reasoning is also able to reveal students' mathematical understanding[15]. The results of a search for 24 articles revealed that there has been no research on multimodal semiotic reasoning of elementary school students in solving two-dimensional shape problems. There was a study limited to semiotic reasoning in students when solving two-dimensional shape problems, but it has not considered students' multimodality[16]. Therefore, research on elementary school students' multimodal semiotic reasoning in solving two-dimensional shape problems is an important novelty to be given in-depth exploration. However, research on semiotic reasoning when students solve

[11] S Kjällander et al., "Elementary Students' First Approach to Computational Thinking and Programming," *Education Sciences* 11, no. 2 (2021): 1–15, https://doi.org/10.3390/educsci11020080; Ifunanya Ubah and Sarah Bansilal, "The Use of Semiotic Representations in Reasoning about Similar Triangles in Euclidean Geometry," *Pythagoras* 40, no. 1 (2019): 1–10, https://doi.org/10.4102/PYTHAGORAS.V40I1.480.

[12] Kjällander et al., "Elementary Students' First Approach to Computational Thinking and Programming."

[13] Ida Bergvall and Anneli Dyrvold, "A Model for Analysing Digital Mathematics Teaching Material from a Social Semiotic Perspective," *Designs for Learning* 13, no. 1 (2021): 1–7, https://doi.org/10.16993/dfl.167; Oti and Crilly, "Immersive 3D Sketching Tools: Implications for Visual Thinking and Communication"; S Pantaleo, "Elementary Students Meaning-Making of the Science Comics Series by First Second," *Education 3-13* 49, no. 8 (2021): 986–99, https://doi.org/10.1080/03004279.2020.1818268.

[14] L Björklund Boistrup and L Gustafsson, "Construing Mathematics-Containing Activities in Adults' Workplace Competences: Analysis of Institutional and Multimodal Aspects," *Adults Learning Mathematics: An International Journal* 9, no. 1 (2014): 7–23.

[15] Deborah Moore-Russo and Janine M. Viglietti, "Using the K 5 Connected Cognition Diagram to Analyze Teachers' Communication and Understanding of Regions in Three-Dimensional Space," *Journal of Mathematical Behavior* 31, no. 2 (2012): 235–51, https://doi.org/10.1016/j.jmathb.2011.12.001; Candace Walkington, Geoffrey Chelule, and Dawn Woods, "Collaborative Gesture as a Case of Extended Mathematical Cognition," no. March 2019 (2019): 1–17; Karina J. Wilkie, "Investigating Secondary Students' Generalization, Graphing, and Construction of Figural Patterns for Making Sense of Quadratic Functions," *Journal of Mathematical Behavior* 54, no. February 2018 (2019): 0–1, https://doi.org/10.1016/j.jmathb.2019.01.005; Xu, Ferguson, and Tytler, "Student Reasoning About the Lever Principle Through Multimodal Representations: A Socio-Semiotic Approach."

[16] C. W. Suryaningrum and Y. D.W.K. Ningtyas, "Multiple Representations in Semiotic Reasoning," *Journal of Physics: Conference Series* 1315, no. 1 (2019), https://doi.org/10.1088/1742-6596/1315/1/012064.

problems from the student's multimodal perspective has never been carried out by other researchers. Therefore, this study aims to analyze students' semiotic reasoning ability when solving two-dimensional shape problems from a multimodal perspective.

RESEARCH METHODS

The research used a qualitative approach and exploratory descriptive type[17]. The instruments consisted of two-dimensional shape essay questions, interview guidelines, and observations. The determination of the research participants was based on the results of multimodal analysis. The total population was 24 third-grade students of SD Muhammadiyah 9 Malang. Multimodal integration can be analyzed using the results of observations, interviews, and written test questions. The guidelines for observations are described in Table 1.

Table 1
Observation Guidelines

Category	Descriptor	Written Answers	Verbal Answers	Gestural Answers	Note
Understanding the Problem	• State the known/identified information • State what is being asked				
Problem Solving Plan	• State the method that will be used to solve the problem • State certain concepts that will be used to solve the problem				
Executing the Problem-Solving Plan	• Use the procedure for implementing the designed plan				
Looking back	• Recheck the procedure that has been followed • Check the suitability of the result for the problem				

[17] John W. Creswell, *Educational Research: Planning, Conducting, and Evaluating Quantitative and Qualitative Research*, 2012; John W. Creswell, *Research Design : Qualitative, Quantitative, and Mixed Methods Approaches*, ed. Vicki Knight et al., *SAGE Publications, Inc.*, 4th ed. (California: SAGE Publications, Inc., 2014).

Category	Descriptor	Written Answers	Verbal Answers	Gestural Answers	Note
Analysis	• Students detect known objects, images, or symbols • Students examine the objects, images, or symbols in question				
Integration	• Students change signs into other representation • Students connect objects, images, or symbols with their previous knowledge • Students connect objects, images, or symbols to solve problems				
Generalization	• Students make a statement that represents the definition of the concept of perimeter of a two-dimensional shape • Students state the procedure for solving the problem of the perimeter of a two-dimensional shape				
Justification	Providing mathematical arguments to support the strategies for solving the problem of the perimeter of a two-dimensional shape				

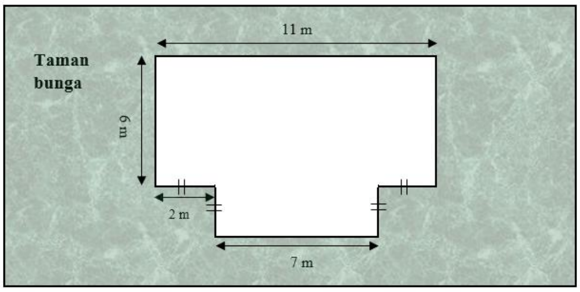
Then interviews were conducted to confirm the answers given by the students and to strengthen the written answers. The guidelines for interviews are presented in Table 2.

Table 2
Interview Guidelines

Category	Descriptor	Note
Understanding the Problem	• Explain the information that you know. • Explain what is being asked.	
Problem Solving Plan	• Explain the method that will be used to solve the problem. • Explain certain concepts that will be used to solve the problem.	
Executing the Problem-Solving Plan	• Explain the procedure for implementing the designed plan.	
Looking back	• What should be done to recheck the procedure that has been followed? • What should be done to check the suitability of the result to the problem situation?	
Analysis	• Describe the known objects, images, or symbols • Explain the objects, images, or symbols in question.	
Integration	• Explain how to convert signs into other representation. • Explain how to connect objects, images, or symbols with previous knowledge. • Explain how to connect objects, images, or symbols to solve problems.	
Generalization	• Make a statement that represents the definition of the concept of perimeter of a two-dimensional shape. • Explain the procedure for solving the problem of perimeter of a two-dimensional shape.	
Justification	• Give mathematical arguments to support the strategies for solving the perimeter problem.	

Two-dimensional shape problems are given according to the Basic Competency (KD) 3.10: Explain and determine the perimeter of two-dimensional shapes. Meanwhile, the indicators determined are indicator 3.10.1: Explain the perimeter of a two-dimensional shape and indicator 3.10.2: Calculate the perimeter of a two-dimensional shape. When solving problems, students are observed for the gestures they make. The result of the problem solving was a written answer. There were 2 participants from the third grade of a private elementary school in the city of Malang. At the initial stage, the students completed a two-dimensional shape problem which was done individually for 15 minutes. The problem given is shown in Figure 1.

Terdapat sebidang kebun berbentuk persegi panjang milik Pak Adi seperti tampak pada gambar di bawah ini. Selama ini, kebun Pak Adi kurang dimanfaatkan sehingga banyak tumbuh tanaman liar. Pak Adi berencana untuk membangun kolam renang dan taman bunga di kebun tersebut. Sketsa kolam renang dan taman bunga tampak pada Gambar di bawah ini.



Untuk memperkirakan biaya yang harus disiapkan, Pak Adi harus menghitung keliling kolam renang. Bantulah Pak Adi untuk menentukan keliling kolam renangnya!

Figure 1
Two-dimensional shape problem

The data from observations, interviews, and written answers was validated to determine the accuracy or credibility of the findings, which was then called validating the findings. Credibility is one of the criteria for the validity/trustworthiness of data in qualitative research[18]. The observation sheets, interview guidelines, and written test questions were validated by lecturers from the Mathematics Education Study Program. The aspects validated in the observation guidelines included (1) the ability to state the indicators of multimodal semiotic reasoning, (2) grammar, (3) openness, and (4) ease of gathering information. Based on the validation results, the average score obtained was agree. This indicated that the result was suitable for use to improve the sentences in the question by simplifying the wording in the test question. The suggestions and input from the validators were then used to improve the instrument. In general, the research followed the procedures shown in Diagram 1.

[18] John W. Creswell and Vicki L. Plano Clark, "Designing and Conducting Mixed Methods Research Approarch," 2018, 849.

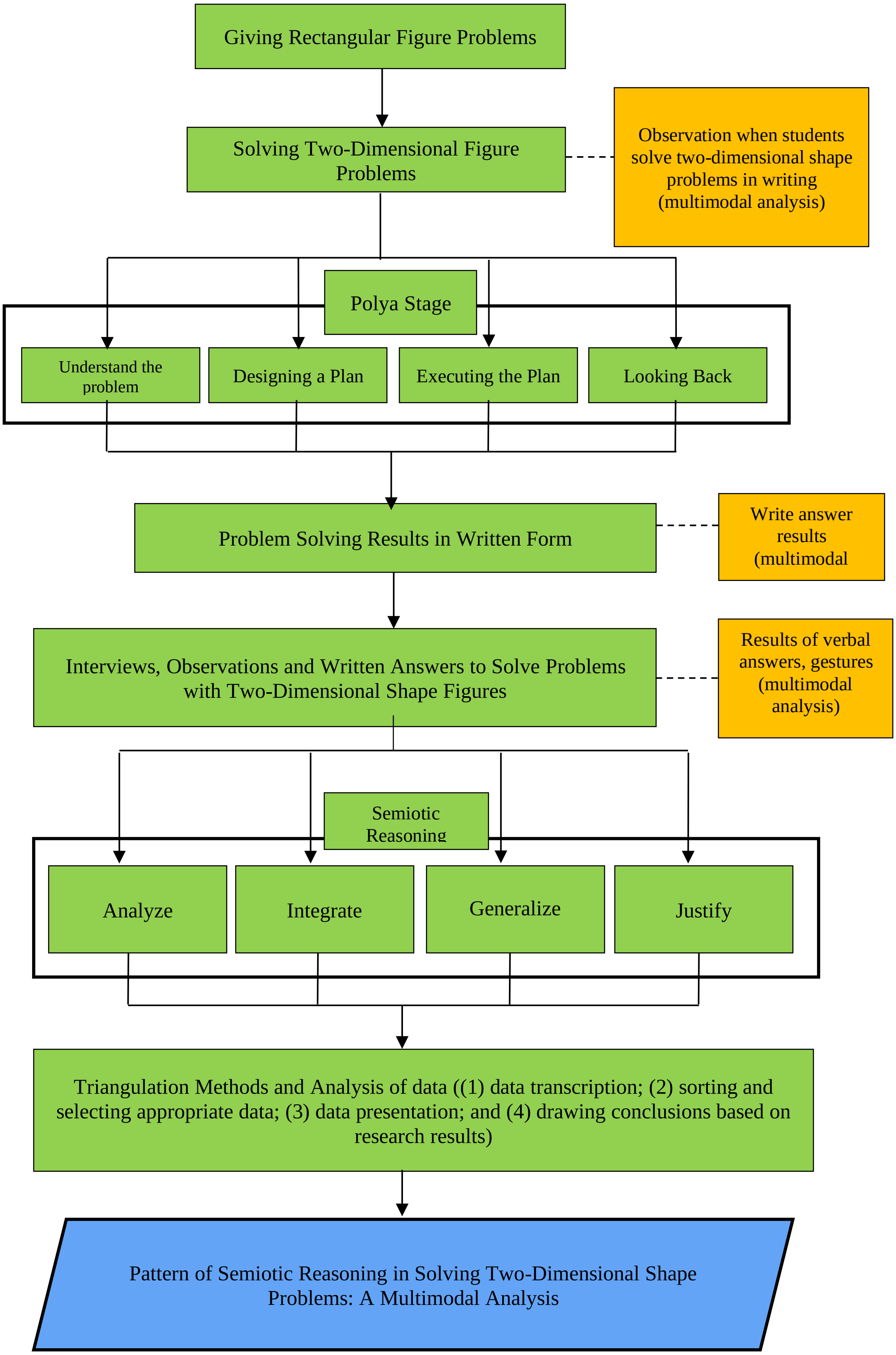


Diagram 1
Procedures of Research

The interview resulted in verbal answers. The data of observations, interviews, and written answers were then analyzed to identify students' multimodality in doing semiotic reasoning to solve two-dimensional shape problems. The data collected identified a) the stages of problem solving done by the students, b) each stage of problem-solving that identified the semiotic reasoning done by the students, and c) the multimodality demonstrated by the students and identified in each indicator of semiotic reasoning. The research analysis used four steps of (1) transcribing the data, (2) sorting and selecting the appropriate data, (3) presenting the data identified based on a) the problem-solving stages according to the Polya's Stages, which include understanding the problem, problem-solving plan, executing the problem-solving plan, and looking back[19], b) the problem-solving stages identified by semiotic reasoning according to the TIMSS 2023 framework (analysis, integration, generalization, and justification)[20], and c) the multimodal identified according to the written, verbal, and gestural answers shown by the students, and (4) drawing conclusions based on the research results.

RESULTS AND DISCUSSION

RESULTS

The results of the member-check and triangulation methods showed that the written answer data, interview results, and observation results were consistent. The results of research on problem-solving activities carried out by the participants were described according to the stages of understanding the problem, planning the problem solving, carrying out the plan, and reviewing based on the written answers, interviews, and observations.

PROBLEM SOLVING BY PARTICIPANT 1 (P1)

Stage 1 Understanding the Problem

The description of the stage of understanding the problem by P1 started from the written answer as shown in Figure 2.

[19] Berinderjeet Kaur, "The Why, What and How of the 'Model' Method: A Tool for Representing and Visualising Relationships When Solving Whole Number Arithmetic Word Problems," *ZDM - Mathematics Education* 51, no. 1 (2019): 151–68, https://doi.org/10.1007/s11858-018-1000-y; S. Adnan, Dwi Juniati, and Raden Sulaiman, "Student's Mathematical Representation in Solving Geometry Problems Based on Cognitive Style," *Journal of Physics: Conference Series* 1417, no. 1 (2019): 1–9, https://doi.org/10.1088/1742-6596/1417/1/012049.

[20] Mullis et al., "Timss 2015 International Results in Mathematics"; Ray Philpot et al., "CHAPTER 1 TIMSS 2023 Mathematics Framework," 2022, 5–18.

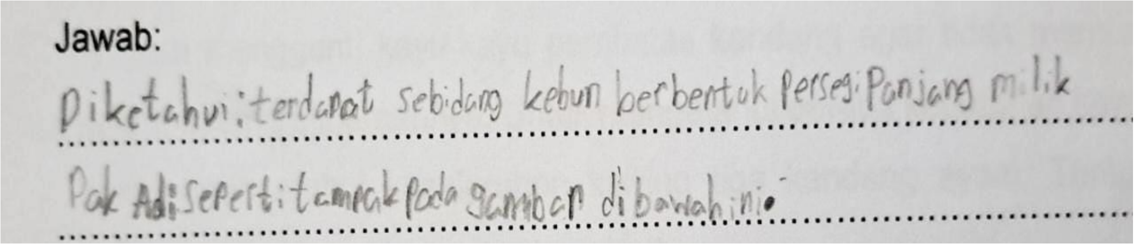

Jawab:
Diketahui: terdapat sebidang kebun berbentuk persegi Panjang milik
Pak Adi: seperti tampak pada gambar di bawah ini.

Figure 2
Result of the written answer to the problem

The answer written in Figure 2 shows that P1 stated the known information “there is a plot of garden with a rectangular shape owned by Mr. Adi as shown in the following figure”. This answer is an indication of the analysis of the problem by P1. The result of the analysis shows the student’s ability to detect known objects, symbols, and signs (interpretants, sign icons). Based on the written answer, an interview session (verbal answer) was conducted as follows.

R: What information does P1 *know about this problem?*

P1: In this number, there is a figure like this (image/object) *(pointing at the picture of a swimming pool)* (gesture/icon). *There is also length and width (indicating the long side and wide side)* (sign icon and symbol)

R: Is there any other information about this problem?

P1: Yes, there is a size (while moving his index finger, pointing at the size of all the sides of the shape and saying this, this) (gesture/sign icon and index)

Based on the results of the interview (verbal answer), P1 added known information in the form of identified image of a shape, length, width, and size of the sides of the shape (indicators of analysis). The results of the observation also showed that the student’s understanding was indicated by the movement of the index finger pointing at the size of all the sides of the shape (gesture) and a verbal answer stating the information about the image of the shape, size, and elements of the shape (symbols, icons). The gesture observed from P1 is shown in Figure 3.

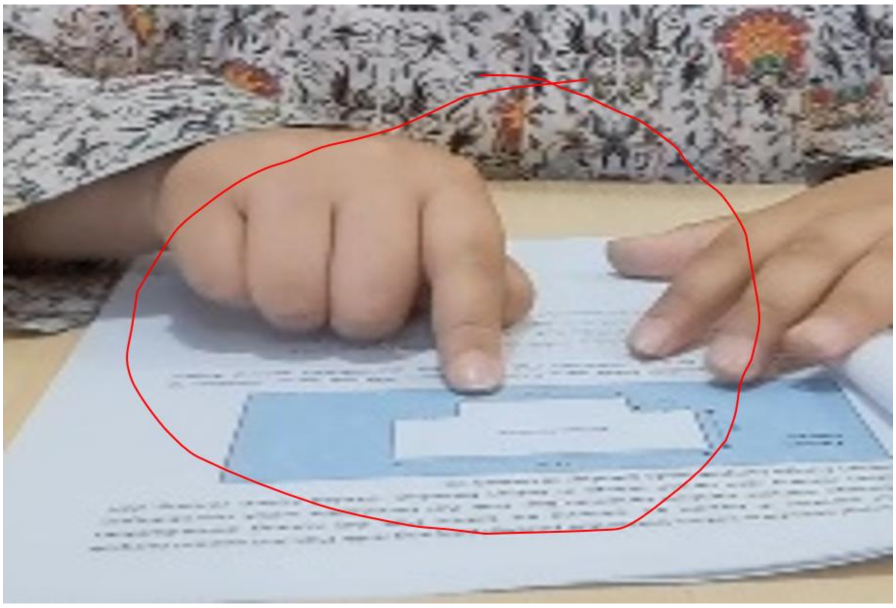


Figure 3
Gesture of P1 during the Interview for the Understanding the Problem Stage

The decision-making process carried out by P1 used indications of analysis for a picture of a swimming pool and involved written, spoken, and gestural forms of multimodality. This was the semiotic reasoning observed from P1's multimodal perspective at the stage of understanding the problem.

Stage 2 Problem-Solving Plan

At the problem-solving planning stage, P1's answers were described based on the results of the interview, observation, and field notes. The result of the interview with P1 can be seen in the following excerpts.

R: What is being asked about this problem?

P1: This problem asks "What is the perimeter of Mr. Adi's swimming pool?" (object, icon sign)

R: How do you solve it?

P1: Well, add all the sides (symbols)

R: Which ones do you want to add?

P1: Well, this plus this plus this (P1 refers to the size of the sides of the swimming pool) (index sign/deictic gesture)

R: On the side where the size is not written, is that also added?

P1: Yes, all the dimensions of the sides are added. (object, sign symbol)

From P1's interview, it was found that his problem-solving plan began with performing indications of analysis as he examined the object, image, or symbols to write down what was being asked. Based on the verbal and gestural answers, P1 showed his problem-solving procedure. In addition, P1 also showed a gesture that expressed the meaning of the perimeter of a shape. These last two indicators showed the process of indication generalization reasoning.

Stage 3 Carrying out Problem Solving

P1 carried out the problem solving as explained in the written answer. During the interview and observation, P1 did not provide any information. The initial step taken by P1 was changing the size of the sides of the shape into a sum form. In this case, the student performed a process of integration indications. The student changed signs into other representation as shown in Figure 4.

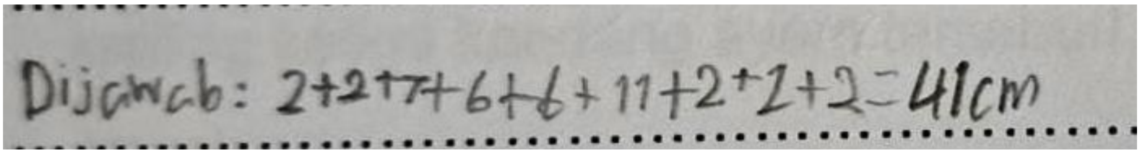


Figure 4
P1 Changing Signs to Other Representation

Changing the sign from the measurements shown on the swimming pool picture into addition involved a “+” (sum) sign. This involvement was part of the integration indication stage, especially the indicator "students connect symbols, signs to solve problems". The result of this addition was the figure "41 cm". The sum result that involved previous knowledge was part of the integration indication activity in the indicator "students connect symbols using previous knowledge". However, P1 gave a wrong answer.

Stage 4 Looking Back

Based on the results of the written answer, interview, observation, and field notes, it was found that the information looked back by P1 began with a multimodal written answer. The result of P1's written answer stated "So, the cost that Mr. Adi has to prepare is 41". This statement showed evidence of the existence of a mathematical argument to support the problem solving in the form of a conclusion marked by the word "so" and the figure "41" as shown in Figure 5.

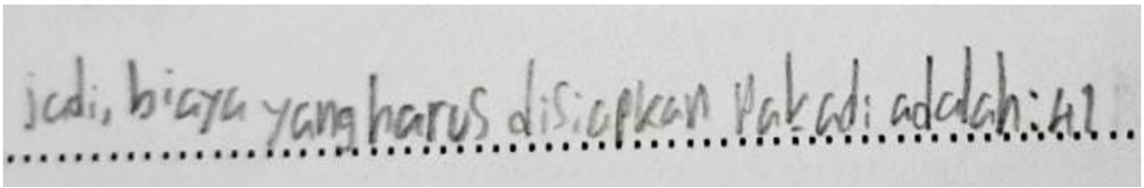


Figure 5
Stage of Looking Back

At this stage of looking back, P1 used an object in the form of Mr. Adi's swimming pool. The object conveyed by P1 are marked using written communication with an icon classification. In addition to signs and object, there were also interpretants by P1 that indicated P1's ability to conclude the solution to the perimeter problem as the result of adding all the sides. This was in accordance with the result of the interview quoted below.

R: Try to read the sentence again "So the cost that Mr. Adi has to prepare is 41". Explain the purpose of writing this sentence! (sign symbol)

P1: Yes, I want to give an answer

R: Where did this answer come from?

P1: Well, from this result (while pointing at the figure 41 from the calculation he made). (sign symbol /deictic gesture)

The review process carried out by P1 contained indications of justification. When reasoning, students involve objects, interpretants, and signs. P1 involved the indexes of signs and icons. Meanwhile, multimodality is based on written, verbal, and gestural answers. From the problem solving carried out by P1, it can be concluded that a reasoning process has occurred, consisting of understanding the problem (indication of analysis), planning the solution to the problem (indication of analysis generalization), carrying out the plan (indication of integration) and looking back (indication of justification). In brief, the pattern can be expressed with the symbols A, A-G, I, J. However, in general, the result of the problem-solving by P1 gave a wrong answer: 41. The error started from stage 3 of problem-solving when he executed his problem-solving plan. P1 added up all the sides with the wrong result. Furthermore, P1 was not so careful as he added one side measuring 2 cm twice. In general, the result of the multimodal analysis carried out by P1 at each Polya’s stage are shown in Table 3.

Table 3
Multimodal Pattern of Analysis, Analysis-Generalization, Integration, and Justification
(Pattern: A, A-G, I, J)

Subject	Component											
	Analysis			Analysis-Generalization			Integration			Justification		
	T	L	G	T	L	G	T	L	G	T	L	G
P1	√	√	√	√	-	-	√	-	-	√	√	√

Problem Solving by Participant 2 (P2)

In this case, the Polya's stages, which include understanding the problem, planning the problem solving, carrying out the plan, and reviewing it, were explained based on the written answer, interview, and observation for P2.

Stage 1 Understanding the Problem

At the initial stage, P2 expressed an understanding of the problem in a written answer as shown in Figure 6.

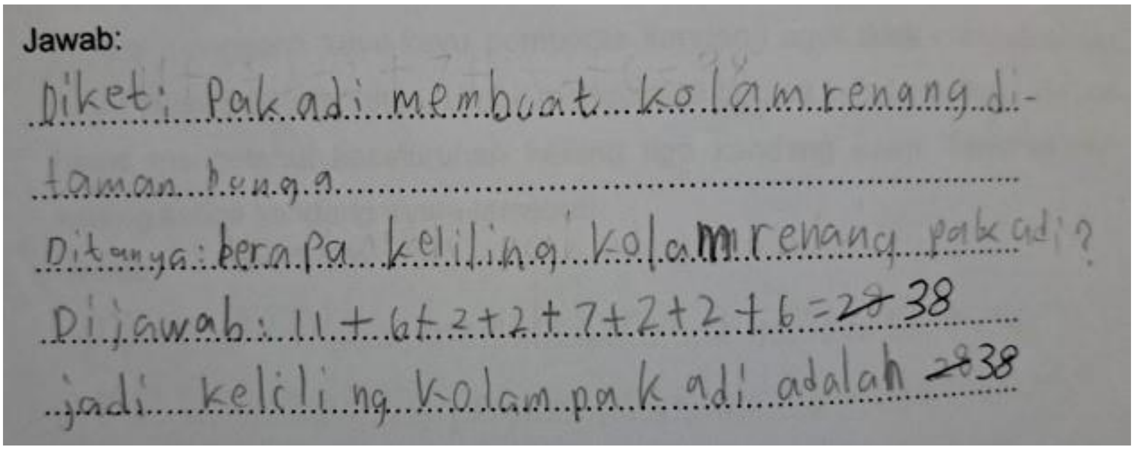

Jawab:
Diket: Pak adi membuat kolam renang di-
taman bunga
Ditanya: berapa keliling kolam renang pak adi?
Dijawab: 11+6+2+2+7+2+2+6 = ~~28~~ 38
jadi keliling kolam pak adi adalah ~~28~~ 38

Figure 6
Result of Written Answer by P2

The answer by P2 shown in Figure 6 contained the information written in the object figure of Mr. Adi's swimming pool. Based on the gesture of P2, he also detected the size of the swimming pool (interpretant, sign icon, symbol). P2's verbal statement is as follows.

R: What information does P2 know in Problem 1?

P2: Mr. Adi's swimming pool (sign icon, object)

R: So, is there any more information shown in question 1?

P2: This one, the size (index sign)

(while pointing his hand at the size of the picture of Mr. Adi's swimming pool)

Based on the result of the interview (verbal answer), P2 added the known information in the form of the size of the sides. The result of this observation showed that P2's deictic and iconic gestures complemented the problem solving. In general, the reasoning stages carried out by P2 showed an indication of analysis.

Stage 2 Problem-Solving Plan

In stage 2 of the problem-solving plan, the answer by P2 was described based on the results the written answer, interview, observation, and field notes. In the written answer, P2 wrote down the question being asked. In addition, it was supported by his verbal answer as shown in the following interview excerpt.

R: What is asked in Problem 2?

P2: Question 1 is "What is the perimeter of Mr. Adi's swimming pool?" (object, icon)

R: Then how do you solve it?

P2: The perimeter is calculated by adding all the dimensions (the sides of the shape). (sign symbol)

The deictic gesture and metaphor used when explaining the problem-solving procedures are shown in Figure 7.

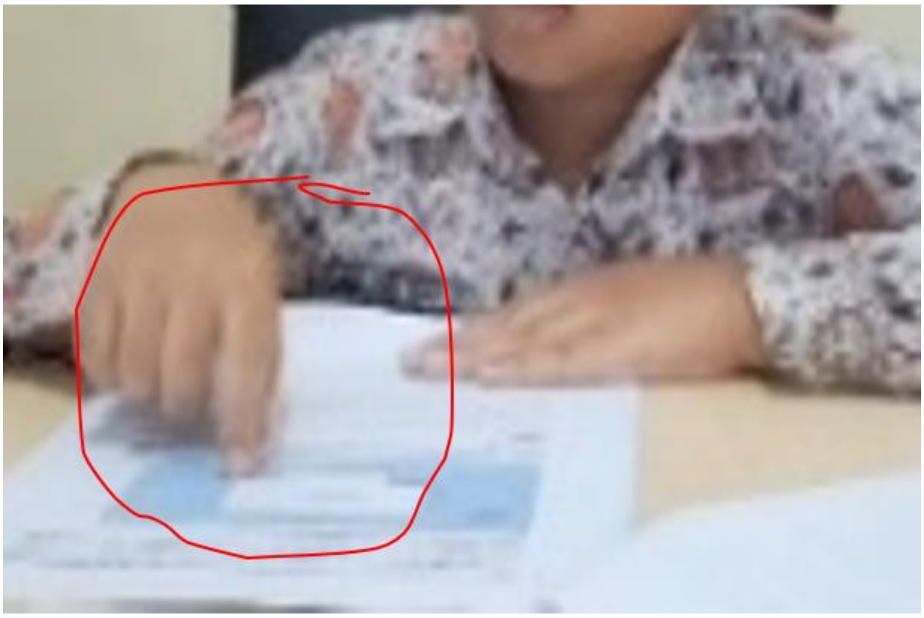


Figure 7
Gesture of P2 When Explaining the Problem-Solving Plan

The results of the written answer, interview, observation, and field notes for P2 showed that the stage of planning for solving the problem began with performing an indication of analysis, in which the student examined the object, image, or symbols to write down what was being asked. Through his gestures, P2 conveyed the procedure for determining the perimeter of the swimming pool (deictic gestures and metaphorical gestures). The procedure presented by P2 was an indication of generalization.

Stage 3 Carrying out the Problem Solving

The subject P2 performed the problem-solving as explained in the written answer. Based on the written answer, it was known that P2 changed the size of the sides of the pool into other representation, namely a mathematical sentence. Then, P2 also connected mathematical sentences using the connector "+" and connected these symbols with previous knowledge. In this case, P2 obtained the sum result, 28, as shown in Figure 8.

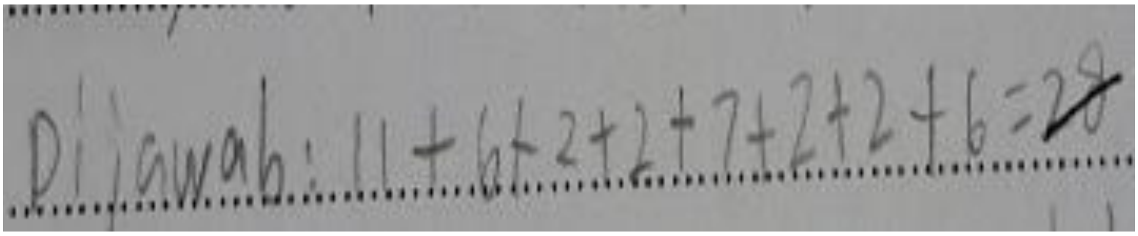


Figure 8
P2's Written Answer When Executing the Problem-Solving

During the interview, P2 followed up on the activity of carrying out his plan by asking permission to correct the answer. P2 felt that there was a calculation error. The researcher eventually allowed him to revise the answer. All the steps carried out in stage 3 of carrying out the problem-solving plan were an indication of integration.

Stage 4 Understanding the Problem

Based on the result of executing the problem-solving plan, P2 said during the interview session that there was a wrong answer. P2 stated that he miscalculated when calculating during the previous stage. Then P2 showed an indication of analysis of the previous calculation. At this stage, P2 indicated an analysis activity, especially for the indicator "students detect identified signs, symbols, objects/images". The result was detecting mathematical sentences, and then the student asked permission to correct his answer. The following figure shows P2's gesture at the stage of understanding the problem.

Figure 9
P2’s Gesture When Understanding the Problem

The semiotic triad occurred with Mr. Adi's swimming pool as the object. The signs used were icons and symbols. Meanwhile, the interpretant was the student's ability to detect identified signs, symbols, and objects/images. The gesture shown by P2 was in the form of a metaphor by presenting abstract ideas of the concept of length, width, and perimeter.

Stage 5 Problem-Solving Plan

P2 planned to recalculate the additions he made. This was because P2 re-analyzed the answer. During the interview, P2 stated that there was a calculation error. The excerpt of the interview is shown below.

R: Why was the answer deleted?

P2: Yes, wrong answer. (sign symbol)

At this stage, P2 carried out a process of reasoning with an indication of analysis, and the multimodal used was verbal multimodal. Then, the semiotic triad occurred with Mr. Adi's swimming pool as the object. The signs used in the form of verbal communication were a type of sign symbol. Meanwhile, the interpretant was the student's ability to connect symbols and signs to solve the problem.

Stage 6 Executing the Problem-Solving Plan

Based on the plan he made, P2 recalculated the problem. He showed an indication of integration with the indicator: students connect symbols and signs using previous knowledge (the ability to add whole numbers). The student performed a calculation as shown in Figure 10.

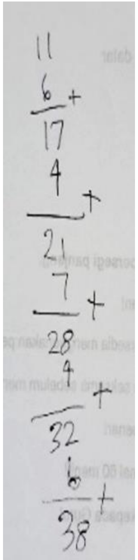


Figure 10
P2’s Answer When Performing Recalculation

The result of the recalculation showed that there was an improvement in the answer, from 28 to 38. At this stage, P2 carried out a reasoning process of integration indication with the multimodal used being written multimodal. Then, there was a semiotic triad using Mr. Adi's swimming pool as the object. The signs used were icons and symbols. Meanwhile, the interpretant was the student’s ability to connect the symbols and signs with previous knowledge, and the student connected symbols and signs to solve the problems and changed the symbols and signs into other representation.

Stage 7 Looking Back

Based on the results of the written answer, interview, observation, and field notes, it was found that the stage of looking back at the information done by P2 came from the multimodal written, verbal, and gestural answers. The result of P2's written answer at the looking-back stage is shown in Figure 11.

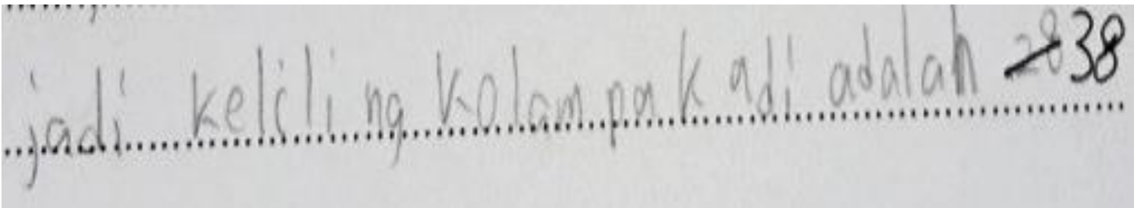


Figure 11
P2's Written Answer When Looking back

P2's answer showed evidence of a mathematical argument to support problem solving in the form of a conclusion marked with the word "so" and a correction of the answer from the figure "28" to "38". At this stage of looking back, P2 used Mr. Adi's swimming pool as the object. The object identified by P2 was marked using written and verbal communication as well as deictic and iconic gestures with the classification of icons and symbols. In addition to the signs and objects, there was an interpretant by P2 in the form of the ability to provide mathematical arguments to support problem-solving.

P2's problem-solving followed the Polya's stages, which include understanding the problem, planning the problem solving, carrying out the plan, understanding the problem, planning the solution to the problem, carrying out the plan, and looking back. The multimodal semiotic reasoning process in solving Problem 2 was identified at each stage of the problem-solving. The reasoning process that occurred was understanding the problem (indication of analysis), problem-solving plan (indication of analysis-generalization), carrying out the problem-solving plan (indication of integration), understanding the problem (indication of analysis), problem-solving plan (indication of integration), carrying out the problem-solving plan (indication of integration) and looking back (indication of justification). In general, the pattern established could be symbolized by A, A-G, I, A, I, I, J. The multimodal used was written, verbal, and gestural answers. The semiotic components in the problem-solving were objects, signs, and interpretants. The object was the picture of Mr. Adi's swimming pool. The signs involved were icons, symbols, and indices. The interpretant was according to each Polya’s stage. The multimodality involved was written, verbal, and gestural answers. In general, the results of the multimodal analysis carried out by P2 at each stage of Polya are shown in Table 4.

Table 4
Multimodal Pattern: Analysis, Analysis-Generalization, Integration, Analysis, Integration, Integration, Justification (Pattern: A, A-G, I, A, I, I, J)

Subject	Analysis			Analysis-Generalization			Integration			Analysis			Integration			Integration			Justification		
	T	L	G	T	L	G	T	L	G	T	L	G	T	L	G	T	L	G	T	L	G
S3	√	√	√	√	√	√	√	√	-	-	-	√	-	√	-	√	-	-	√	√	√

The Polya's stages of problem solving were carried out sequentially, and no stages were skipped. The stages can be carried out in different orders, but all the stages must be carried out properly[21] because if there are missing stages, it can affect the problem solving. However, the indications of semiotic reasoning at each of these stages need in-depth discussions.

DISCUSSION

Based on the results of the analysis of the two participants, different problem-solving patterns were found. In the problem-solving process, both participants followed the Polya's stages. According to Polya, the stages include understanding the problem, planning problem-solving, implementing the problem-solving plan, and looking back[22]. Polya did not specify that these stages must be sequential, but Polya stated that each stage starts from the stage of understanding the problem[23].

When the participants solved the problem, two semiotic reasoning patterns were obtained from both participants. The first pattern carried out by P1 was analysis, analysis-generalization, integration, and justification (A, A-G, I, J). This pattern indicated that at the stage of understanding the problem, P1 performed an analysis indication. Then, at the problem-solving planning stage, P1 showed an indication of analysis followed by an indication of generalization. P1 performed two reasoning frameworks at the problem-

[21] John H. Conway, "How to Solve It A New Aspect of Mathematical Method G. Polya.Pdf" (Priceton University Press, 2004).

[22] Adnan, Juniati, and Sulaiman, "Student's Mathematical Representation in Solving Geometry Problems Based on Cognitive Style"; Kaur, "The Why, What and How of the 'Model' Method: A Tool for Representing and Visualising Relationships When Solving Whole Number Arithmetic Word Problems."

[23] S. H. Kaliky, E. Nurlaelah, and A. Jupri, "Analysis of Mathematical Problem Solving Ability Students of Junior High School to Polya Model," *Journal of Physics: Conference Series* 1157, no. 4 (2019): 2–5, https://doi.org/10.1088/1742-6596/1157/4/042064; A. L. Son, Darhim, and S. Fatimah, "An Analysis to Student Error of Algebraic Problem Solving Based on Polya and Newman Theory," *Journal of Physics: Conference Series* 1315, no. 1 (2019), https://doi.org/10.1088/1742-6596/1315/1/012069.

solving planning stage, including the analysis and generalization. At the stage of implementing the problem-solving plan, P1 showed an indication of integration. Then, the problem solving by P1 ended with a review or looking back marked by an indication of justification by P1.

The other pattern was carried out by P2, consisting of analysis, analysis-generalization, integration, analysis, integration, integration, and justification (A, A-G, I, A, I, I, J). This pattern indicates that the problem-solving according to the Polya's stages was not carried out sequentially. The research on P2 showed that the problem-solving was carried out through the stages of understanding the problem, planning the problem solving, executing the problem-solving plan, understanding the problem, planning the problem solving, carrying out the problem-solving plan, and looking back. The pattern A, A-G, I, A, I, I, J in P2's problem solving indicated that at the stage of understanding the problem, P2 showed an indication of analysis. Meanwhile, in the planning stage for solving the problem, P2 showed an indication of analysis continued with generalization. P2 showed an indication of reasoning framework at the stage of problem-solving planning, namely analysis and generalization. When carrying out the problem-solving plan, P2 showed an indication of integration. Then, P2 returned to understanding the problem, which reflected an indication of analysis. Then, P2's planning to solve the problem was an indication of integration. When executing the problem-solving plan, P2 again showed integration. Meanwhile, the final stage of problem-solving, the looking back, was an indication of a justification activity by P2.

These two patterns indicate that the participants used semiotic reasoning indicators when solving a problem. In future research, there may be other patterns. Students are free to follow Polya's stages in solving problems, and they do not have to be in order[24]. Similarly, the reasoning components according to TIMSS 2023 can be done according to students' convenience, and they do not have to be in order[25]. Both patterns show that all indicators of reasoning have appeared in the students' answers. However,

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[24] John H. Conway, "How to Solve It A New Aspect of Mathematical Method G. Polya.Pdf"; D. Fahrudin, Mardiyana, and I. Pramudya, "The Analysis of Mathematic Problem Solving Ability by Polya Steps on Material Trigonometric Reviewed from Self-Regulated Learning," *Journal of Physics: Conference Series* 1254, no. 1 (2019): 1–7, https://doi.org/10.1088/1742-6596/1254/1/012076.

[25] Martin and Davier, *TIMSS 2023 Assessment Frameworks TIMSS 2023 Assessment Frameworks*; Philpot et al., "CHAPTER 1 TIMSS 2023 Mathematics Framework."

the quality needs further in-depth research through different perspectives in mathematics learning.

Investigation of indications of semiotic reasoning in the Polya's stages can be improved based on the multimodality of participants. P1 used multimodal answers in the form of written, verbal, and gestural answers when carrying out analysis indications at the stage of understanding the problem and justification indications at the stage of looking back. Meanwhile, in the other indications of semiotic reasoning, P1 used a written answer without using other answer types. On the other hand, the multimodal was used by P2 at the stages of understanding the problem, planning the problem solving, and looking back. In other semiotic reasoning processes, P2 used written, verbal, and gestural answers.

Each stage of problem-solving gives rise to a semiotic reasoning component based on students' multimodality[26]. Multimodality has represented the results of mathematical activities, with the potential to influence students' thinking to understand the meaning of the perimeter of a rectangle more generally[27]. The multimodal engagement produced by students is an important aspect of solving two-dimensional shape problems [28]. There is a correlation between different sources, including written, verbal, and gestural answers[29]. Different ways of each resource are used to express students' thinking. When examining interactions between resources, several possibilities occur. The existence of different resources provides the same information. For example, in written and verbal answers,

[26] Lotherington Heather, "Elementary School Language and Literacy Education for Civic Engagement: An Evolving Playbook for Postmodern Times," *Language and Literacy* 19, no. 3 (2017): 4; K Drotner, "Children's Digital Content Creation: Towards a Processual Understanding of Media Production among Danish Children," *Journal of Children and Media* 14, no. 2 (2020): 221–36, https://doi.org/10.1080/17482798.2019.1701056.

[27] E Andriyanti, "Social Meanings in School Linguistic Landscape: A Geosemiotic Approach," *Kemanusiaan* 28, no. 2 (2021): 105–34, https://doi.org/10.21315/KAJH2021.28.2.5; J Bezemer and S Abdullahi, "Multimodality," in *The Routledge Handbook of Linguistic Ethnography* (Centre for Multimodal Research, UCL Institute of Education, United Kingdom: Taylor and Francis, 2019), 125–37, https://doi.org/10.4324/9781315675824-10; M M W Cheng, K Danielsson, and A M Y Lin, "Resolving Puzzling Phenomena by the Simple Particle Model: Examining Thematic Patterns of Multimodal Learning and Teaching," *Learning: Research and Practice* 6, no. 1 (2020): 70–87, https://doi.org/10.1080/23735082.2020.1750675.

[28] Heather, "Elementary School Language and Literacy Education for Civic Engagement: An Evolving Playbook for Postmodern Times"; L Rincon-Mendoza, "Beyond Language Fluidity: The Role of Spatial Repertoires in Translingual Practices and Stancetaking," *International Journal of Bilingual Education and Bilingualism*, 2020, https://doi.org/10.1080/13670050.2020.1775780.

[29] W Toh and F V Lim, "Using Video Games for Learning: Developing a Metalanguage for Digital Play," *Games and Culture* 16, no. 5 (2021): 583–610, https://doi.org/10.1177/1555412020921339; E A Kozhemyakin and V G Lovyagina, "Targeted advertising in social networks: Searching for efficient semiotic models," *Vestnik Moskovskogo Universiteta. Seriya 10. Zhurnalistika* 2020, no. 5 (2020): 3–28, https://doi.org/10.30547/vestnik.journ.5.2020.328.

students provide the same information. Alternatively, different resources provide contradictory information as what happens when students correct their calculation results. In this case, there are differences among written, verbal, and gestural answers[30]. Furthermore, there is a possibility that, in students' multimodal processes, there will be complementary processes. Differences in information or contradictions has an impact on the reasoning process performed[31]. The impact is in the form of variations in the reasoning patterns carried out by the participants. Therefore, in solving two-dimensional shape problems in the classroom, teachers need to explore step by step the reasoning process carried out by students in accordance with the multimodality they use. In addition, the discussion of problem-solving topic done by the two participants shows that the two patterns do not necessarily lead to the correct answer. The first pattern, A, A-G, I, J, led P1 to give the wrong answer. Future research needs to conduct an in-depth investigation of students' error in problem-solving. In this case, participant 1 gave the wrong answer not because he had not carried out all Polya's stages or semiotic reasoning but because there was a wrong process when carrying out integration at the stage of executing the problem-solving plan. Therefore, it is necessary to do further research that can improve students' reasoning ability in solving mathematical problems.

CONCLUSION

There were two patterns of problem-solving. The first pattern was understanding the problem, planning the solution to the problem, implementing the plan, and looking back. The second problem-solving pattern was understanding the problem, planning the problem-solving, implementing the problem-solving plan, understanding the problem, planning the problem-solving, implementing the problem-solving plan, and looking back. The first pattern of semiotic reasoning was analysis, analysis-generalization, integration,

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[30] Betül Çimenli, "On Pronunciation Teaching and Semiotics," *Procedia - Social and Behavioral Sciences* 199 (2015): 634–40, https://doi.org/10.1016/j.sbspro.2015.07.590; Núria Planas, Candia Morgan, and Marcus Schütte, "Mathematics Education and Language: Lessons and Directions from Two Decades of Research," *Developing Research in Mathematics Education. Twenty Years of Communication, Cooperation and Collaboration in Europe* 38, no. 3 (2018): 196–210.

[31] Melanie Williams, Kok-Sing Tang, and Mihye Won, "ELL's Science Meaning Making in Multimodal Inquiry: A Case-Study in a Hong Kong Bilingual School," *Asia-Pacific Science Education* 5, no. 1 (2019): 1–36, https://doi.org/10.1186/s41029-019-0031-1; Çimenli, "On Pronunciation Teaching and Semiotics"; Ellen Beate Hellne-Halvorsen, Leif Christian Lahn, and Hæge Nore, "Writing Competences in Norwegian Vocational Education and Training: - How Students and Apprentices Express Their Professional Competences," *Vocations and Learning* 14, no. 2 (2021): 243–64, https://doi.org/10.1007/s12186-020-09262-0.

and justification (A, A-G, I, J) while the other pattern consisted of analysis, analysis-generalization, integration, analysis, integration, integration, and justification (A, A-G, I, A, I, I, J). The multimodal used was in the form of written, verbal, and gestural answers when performing the analysis indications at the stage of understanding the problem and justification indications at the stage of looking back. In the first pattern, the participant gave a wrong answer not because he had not carried out all Polya's stages of reasoning or semiotics but because there was a wrong process when carrying out integration at the stage of implementing the problem-solving plan. Therefore, it is necessary to conduct further research that can improve students' reasoning ability when solving mathematical problems. Several further research opportunities that can be carried out include analyzing students' errors at each stage of reasoning and research on misconceptions about students' semiotic reasoning in solving mathematical problems.

ACKNOWLEDGMENT

The authors would like to thank the Basic Education Study Program of the Postgraduate School of Malang State University as a place for the doctoral studies.

DECLARATION OF CONFLICTING INTERESTS

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FUNDING

The research funding comes from the Block Grant funds from the Faculty of Teacher Training and Education of Muhamamdiyah University of Malang.

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