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Research article

Education

LEVELS OF CONTROVERSIAL REASONING OF THE PRE-SERVICE TEACHERS TO SOLVE MATHEMATICAL PROBLEMS

職前教師解數學題的爭議推理水平

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Abstract

Controversial reasoning is very important to learn because controversial problems often occur in everyday life. This study examines controversial reasoning in the context of solving mathematical problems. Therefore, this study aims to develop levels of controversial reasoning in solving mathematical problems. The subjects of this study were 185 mathematics education undergraduate students in the sixth and eighth semesters as prospective mathematics teachers. Most of them had taken courses in Pedagogy and Mathematics. The students were given three controversial mathematical problems and interviewed in-depth. The students' thought processes in solving problems construct and determine the characteristics of controversial mathematical reasoning. The study found three controversial mathematical reasoning levels characteristics: initial, exploration, and clarification. At the initial level, the subjects can recognize the controversy on an issue but cannot trace the cause of the controversy itself. At the exploration level, the subjects can explore the components that cause the problem to become controversial but cannot clarify the problem as a solution. At the level of clarification, the subjects can clarify controversial issues using plausible reasons. All the 185 subjects who answered show the answers that can be classified into three levels: initial 55 (29.73%), exploration 52 (28.11%), and clarification 78 (42.16%) subjects.

Keywords: Controversial Reasoning Level, Problem-Solving, Pre-Service Teachers

摘要 有爭議的推理是非常重要的學習，因為有爭議的問題經常發生在日常生活中。本研究在解決數學問題的背景下檢查有爭議的推理。因此，本研究旨在培養解決數學問題的有爭議的推理水平。本研究的對像是 185 名數學教育本科生在第 6 和第 8 學期作為未來的數學教師。他們中的大

多數人都參加過教育學和數學課程。學生們接受了三個有爭議的數學問題，並進行了深入採訪。學生在解決問題時的思維過程被用來構建和確定有爭議的數學推理特徵的水平。研究發現了有爭議的數學推理層次的三個特徵，即初始、探索和澄清。在初始層面，被試可以識別一個問題上的爭議，但無法追查爭議本身的原因。在探索層面，受試者可以探索導致問題成為爭議的組成部分，但不能將問題作為解決方案加以澄清。在澄清層面，受試者可以使用合理的理由澄清有爭議的問題。所有回答的 185 名受試者給出的答案可以分為三個級別：初始 55（29.73%）、探索 52（28.11%）和澄清 78（42.16%）受試者。

关键词: 有爭議的推理水平，解決問題，職前教師

I. Introduction

Learning mathematics is a process of developing thinking skills that can be used to solve problems. Mathematical thinking skills and meaningful mathematical understanding are among the goals of current mathematics education [17]. Problem-solving becomes the core in learning mathematics, which has been set by [28] as one of the five standards of the mathematics learning process. There are five standard processes formulated by [28]: problem-solving, proofs and reasoning, communication, connection, and representation. The importance of problem-solving in learning mathematics encourages many researchers to study problem-solving [13, 30] when teacher-student interaction occurs. [30] explain that problem-solving is not only influenced by mathematics learning but also influenced by psychological aspects.

Mathematical reasoning becomes one of the most important parts of learning mathematics [6, 24, 25, 28, 35]. [35] examine the student's reasoning in learning mathematical proofs and the development of student reasoning through learning mathematics, which is explained further that reasoning is a process that allows one to recall ideas and knowledge obtained to build new arguments. [35] explain that reasoning refers to the activities of thinking that involve giving reasons that make sense in solving variation. [6] discuss reasoning related to obstacles and cognitive support in understanding integers. [26] assert that reasoning becomes the foundation in problem-solving. Many studies of reasoning show that reasoning is important in learning mathematics. [28] has included reasoning as one of the standards that became the goal of learning mathematics.

The study of reasoning has entered into a variety of mathematical content, so that the terms quantitative reasoning, co-variational reasoning, proportional reasoning, statistical reasoning, algebraic reasoning, etc., where mathematical content becomes a character in reasoning. This study is different from the previous ones because it examines the controversial reasoning using the contents of controversial mathematical problems.

The controversial problems are the circumstances that cause debate because of different points of view. A thought is called controversial if it differs from general opinions. A controversy can occur when someone encounters a different problem from the commonly considered normal problem in mathematics. Controversial problems stimulate debate because there are differences between the usual conditions. In everyday life, it is often encountered as a matter of controversy. Controversial problems arise due to an understanding of an issue that has not been completed, causing conflicts in one's thinking. Conflicts of thought encourage one to study more deeply related to the problem and ultimately raise various arguments that can support/strengthen/change their opinions or otherwise influence others to change opinions so that they follow themselves. In dealing with controversial problems, a person needs a logical argument to find the components of the problem and can provide a reasonable reason for the problem at hand. For example, the productive age population in a country in the next 15 years is more than that of non-productive age. Experts conclude that the country will receive a demographic bonus. However, many people who have different opinion causes the condition of becoming a demographic disaster. At first glance, it can be recognized that these two things are controversial. The reasons for each of these opinions can be explored more. The demographic bonus can occur if the productive age population can work optimally, and like the effect, it will increase the state income.

On the other hand, demographic disasters will appear for many reasons, such as political instability and threats to security or management that cannot provide jobs. This illustration shows that a condition becomes a bonus or disaster, really depends on the reasons. Therefore, logical reasoning is needed in solving a controversial problem, referred to as controversial reasoning.

Several researchers [26, 32] have studied controversial reasoning. [26] examines students' reasons when solving problems related to controversial arguments. In this case, the controversial argument is focused on invalid arguments. Students are confronted with the problems of a wrong controversy and are asked to argue with reasons. [32] examined the reasoning of students in dealing with controversial problems related to socio-scientific and global problems. The research results show that student reasoning varies according to the problem, specifically because of their emotional closeness and socio-cultural origins. This shows that the controversial reasoning is closely related to the students' experience or knowledge construction scheme. In the context of mathematics, controversial reasoning can occur because there is a difference between mathematical knowledge possessed (already felt following general truth) and the problem at hand. The problems encountered by someone are different from the truth-values that they have constructed.

A controversial problem triggers someone to recognize the existence of controversy or contradiction, explore the components that cause controversy/ contradiction, and clarify. The earliest process (initial) in solving the problem of controversy is to recognize. People who can recognize the controversy may not necessarily explore the components that cause the controversy. He probably could only feel the controversy with the knowledge he had. Someone who can explore the components of a problem has a higher level than someone who can only recognize.

Furthermore, the people who can explore the components cannot necessarily clearly provide solutions to the controversy. On the other hand, someone who can clarify can certainly explore because the clarification process is based on exploration. Based on the explanation above, it can be concluded that the level of general controversial reasoning starts from the initial (recognizing), exploration, and clarification, which are referred to as the levels of general controversial reasoning.

Controversial problems are very much related to the problem-solving process. In this research, problem-solving is focused on mathematical problems. Problem-solving is one of the main aspects of the mathematics curriculum, which requires students to apply several mathematical concepts and skills [38]. The mathematical problem-solving itself can be seen from the perspective of cognitive conflict. Controversial reasoning begins with cognitive conflict in dealing with problems. There are several studies [10, 11, 22] that have discussed cognitive conflict. According to [10], cognitive conflict in mathematics relates to understanding and problem-solving. According to [22], cognitive conflict is influenced by one's point of view in understanding a problem.

There are several studies on cognitive conflict, among others [21, 40, 41]. According to [41], when students are presented with a controversial problem, a different point of view will emerge that affects what has been previously understood. Furthermore, [40] explain that students do not intend to make different points of view, but different views are formed to comprehend the problem as a whole. Cognitive conflict is also influenced by the source's credibility and context, especially when looking at the problem. [21] explain that in dealing with a controversial problem, a different reasoning pattern would occur.

Someone who experiences cognitive conflict can indirectly develop their critical thinking skills [15]. In this situation, there is a conflict between the knowledge possessed by students and the situation at hand. When there is a cognitive conflict, someone can think critically. This shows that cognitive conflict in students can bring up critical thinking skills. The importance of critical thinking as one of the students' thinking skills has led to various studies on critical thinking skills [3, 4, 42], critical thinking disposition [2, 5, 18], and critical thinking assessment [39].

Teachers in classes must improve their questioning ability in teaching-learning processes. Higher-order questions, which can promote critical thinking, were infrequently used during teaching [7]. Teachers' perceived academic emphasis was commonly associated with teachers who claimed to provide high-quality mathematics instruction with high self-efficacy [34].

In dealing with problems, someone who experiences cognitive conflict will reflect and continue to criticize the problem. This shows a relationship between cognitive conflict, reflective thinking, and critical thinking to resolve controversial problems. Several studies discuss the relationship between reflective thinking and critical thinking skills [12, 14, 33]. [14] explain that reflective thinking is one important factor in solving a problem and can cause critical thinking. [12] also examined the relationship between critical thinking skills and reflective thinking.

The results show that critical and reflective thinking are significantly positively correlated.

Reflective thinking is an important thing that needs to be considered in learning mathematics, especially in solving mathematical problems [23]. Reflective thinking is one of the high-level thinking skills that students must possess [16]. Pre-service teachers and students also need reflective thinking. The development of reflective thinking influences the effectiveness of learning in learning practices [29]. The tendency of reflective thinking of pre-service mathematics teachers is very helpful in learning and teaching mathematics [20]. Reflective thinking can be identified from the phases of learning and metacognitive activities [27]. Students' reflective thinking skills are important in problem-solving and are influenced by gender [9]. Prospective teacher's conceptual understanding, critical thinking, problem-solving, and mathematical communication skills are all found to be significantly related in reflective learning groups [19]. There are four main categories used to analyze reflective thinking in solving problems: (1) the formulation and synthesis of experiences, (2) the regularity of experience, (3) evaluation of experience, and (4) testing solutions chosen based on experience [1]. This means that in dealing with controversial problems, students will experience cognitive conflict and solve the controversial problems using critical and reflective skills.

II. Method

This research used a sequential mixed-methods exploratory sequential design, beginning with a qualitative design and continuing with a quantitative design. The qualitative design is used to explore levels of controversial reasoning, and the quantitative design is used to assess the distribution of levels based on semester (semester 6 / semester 8) and gender (male/female). This study involved 200 student candidates for semester 6 (43%) and semester 8 (57%) in the mathematics department at two universities in Malang, namely 54% of a State University (SU) and 46% of a Private University (PU). They are all pre-service teacher candidates for Mathematics. They will become Mathematics teachers if they have taken all the courses in the Mathematics education curriculum. The curriculum consists of 54.20% Mathematics, 35.11% pedagogical abilities, 10.69% general knowledge. They have taken many courses, including basic mathematics, statistical methods, mathematical statistics 1, real analysis, algebraic structures, number theory, and calculus. The following table is shown the distribution of subjects based on their universities.

Table 1.
Distribution of subjects based on universities

University	6th semester		8th semester	
	Male	Female	Male	Female
SU	23	25	28	32
PU	18	20	26	28

The instrument in this study was a written test in which there were three problems. The first and second problems are related to the simplification of algebraic forms. The third problem is related to the combined operation of roots and squares. The first problem is an implicit controversial problem, where the controversy is not immediately apparent. In contrast, the second and third problems are explicitly controversial problems, where the controversy is immediately apparent and where the process seems reasonable, but the results are contradictory. Three questions were made because they were used as triangulation to obtain consistent data from controversial levels of reasoning.

Problem 1: The Form of Algebra (Implicitly Controversial)

When the teacher gives questions to students, simplify the algebraic form $\frac{2x^2-3xy-2y^2}{x-2y}$. *Students solve it by factoring in the numerator and dividing the same shape as the denominator.*

$$\frac{2x^2-3xy-2y^2}{x-2y} = \frac{\cancel{(x-2y)}(2x+y)}{\cancel{(x-2y)}} = 2x + y$$

Other students ask, "Can (x-2y) be divided by (x-2y)? How about x = 2y? "

a. Do you think that the student's question makes sense? Explain that!

b. If you were the student's teacher, what can you explain the problem to understand it well?

Problem 2: The Form of Algebra (Explicitly Controversial)

A teacher gives questions to students as follows.

"Given that $a^2 - b^2 = (a+b)(a-b)$*. If a = b, then simplify the equation!".*

Students answer as follows.

$a^2-a^2 = (a+a)\,(a-a)$

$a(a-a) = (a+a)(a-a)$,

$a\,(a-a) = (a+a)\,(a-a)$, *divided both sides by (a-a)*

$a = a+a$

$a = 2a$

$a = 2a$, *divided both side by a*

$1 = 2$

a. In your opinion, does the student's answer make sense? Explain that!

b. If you were the student's teacher, what can you explain the problem to understand it well?

Problem 3: Root and Rank Operations (Explicitly Controversial)

A teacher gives questions to students: $\sqrt{(-2)^2}$ *=.....*

Two students (S1 and S2) answer differently as follows.

S1 answered: $\sqrt{(-2)^2}$ *= 2 with reason* $\sqrt{(-2)^2} = \sqrt{4} = 2$*, while S2 answered:* $\sqrt{(-2)^2}$ *= -2 with reason:* $\sqrt{(-2)^2} = ((-2)^2)^{\frac{1}{2}} = (-2)^1 = -2$*.*

a. Do you think the answers of the two students make sense? Explain that!

b. If you were the teacher of the two students, what can you explain about the problem so that both students can understand well?

III. Results and Discussion

Before the controversial reasoning is examined, the completeness and consistency of the subjects' answers were tested first. After the three problems were given to the subjects, the answers were compared between the first problem, the second problem, and the third problem. Subjects who answered in full and consistent thought patterns continued with the grouping of controversial reasoning. Of the 200 subjects, 185 (92.5%) people wrote complete and consistent answers. Whereas 15 (7.5%) people answered incomplete or inconsistent questions. The subjects who answered completely and consistently were interviewed in-depth to determine the category of controversial reasoning. The data and controversial reasoning categorization analysis identified three levels, initial, exploration, and clarification. Of the 185 subjects, there were 38 (29.73%) people at the initial level, where the subjects caught the contradiction but did not know the components that caused the contradiction. Fifty-two (28.11%) people are at the level of exploration. The subjects were able to catch contradictions and explore the components of the problem that cause contradictions but could not find a solution. As many as 78 (42.16%) people are at the level of clarification. The subjects were able to perceive contradictions, explore and find mathematical solutions logically, and explain various reasons that can be used to justify solutions. Table 2 presents the distribution of subjects by the controversial level of reasoning and gender.

Table 2.
Distribution of subjects based on controversial reasoning and gender

Gender	Controversial Reasoning Level			Total
	Initial	Exploration	Clarification	
Male	38 (20.54%)	25 (13.51%)	35 (18.92%)	98 (52.97%)
Female	17 (9.19%)	27 (14.59%)	43 (23.24%)	87 (47.03%)

Table 2 shows that the controversial reasoning at the initial level is more dominated by male subjects, the balanced level of exploitation, and the clarification level is more dominated by female subjects.

A further detailed discussion regarding each level of controversial reasoning is presented as follows.

A. Initial Level

Fifty-five (29.73%) subjects were at the initial level with a distribution of 38 (20.54%) men and 17 (9.19%) women. At this level, the subjects were able to grasp the contradiction but unable to grasp the components that caused the contradiction and were unable to obtain the correct solution. At this initial level, it is more dominated by male subjects. The subjects' behaviors in solving problems and the controversial level of reasoning characteristics of the initial level are presented in Table 3.

Based on the subjects' behaviors in solving problems, written answers, and the results of the interviews in Table 3, we can get the controversial level of initial reasoning characteristics that the subjects begin to recognize the existence of controversy (contradiction) between the facts faced and the knowledge that they have already had. Even contradictions are strengthened by procedures that have been constructed considered as common and different facts so that cognitive conflict arises. However, the subjects were not able to continue the process of finding the components that cause contradictions. The emerging cognitive conflict [8], [26], [41] could be developed into learning to increase understanding.

B. Explorative Level

At this level, the research subjects have been able to grasp the contradiction and trace the components of the problem that cause the contradiction but were unable to produce the right solution. Fifty-two (28.11%) subjects were at the exploratory level. At this explorative level, it is balanced between male and female subjects. The subjects' behaviors in solving problems and the characteristics of the controversial exploratory level of reasoning are presented in the following Table 4.

C. Explorative Level

At this level, the research subjects have been able to grasp the contradiction and trace the components of the problem that cause the contradiction but were unable to produce the right solution. Fifty-two (28.11%) subjects were at the exploratory level. At this explorative level, it is balanced between male and female subjects. The subjects' behaviors in solving problems and the characteristics of the controversial exploratory level of reasoning are presented in the following Table 4.

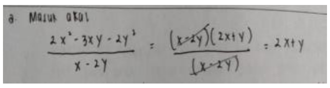
Based on the subject's behaviors in solving problems, written answers, and the results of interviews in Table 4, we can obtain the controversial level of controversial reasoning characteristics that the subjects recognize contradictions and can continue identifying the components that cause contradictions. However, the subjects have not been able to continue the process of reasoning that produces correct answers. The subjects have given rise to a vague concept of $\frac{0}{0}$, but no solution has been offered yet. Subjects are still dominated by procedural knowledge [31].

D. Clarification Level

At this level, research subjects can make mathematical, logical solutions or explain various reasons that can be used to justify solutions to contradictions. Seventy-eight (42.16%) subjects were at this level. Female subjects mostly dominate at this clarification level. The subjects' behaviors in solving problems and the controversial level of reasoning characteristics of reasoning are presented in Table 5.

Based on the subject's behaviors in solving problems, written answers, and the results of interviews in Table 5, the characteristics of the controversial level of reasoning clarification can be obtained. The subject can clarify the components and controversial sources and make solutions logically mathematic or explain various reasons that can be used to justify solutions from contradictions. In this case, solving this controversial problem, the subject can clarify and use the concept that is owned well, finally producing the right solution [31, 37].

Table 3.
Subjects' behaviors at initial level

No	Subjects Behavior	Answer of Subjects	The Characteristics of Initial level
1.	Stating that what students say makes sense, for x = 2y will result in x - 2y is 0 Whereas the division of zero by zero is not permissible, it is contrary to the law of chancellery.	Implicit Controversial  Copy of the subject's answer a. Make sense that $\frac{2x^2-3xy-2y^2}{x-2y}=\frac{(x-2y)(2x+y)}{(x-2y)}=2$ Interview Q: Why is it divided by (x-2y)? S1: Because the values are the same	Recognize a contradiction but do not know the components that cause the contradiction. This controversial issue is explicit because it is not immediately apparent. The new controversy appears when division occurs with zero. The subjects mentioned may be divided by (x-2y) if the form of multiplication.
2.	State that when working on the problem from only one direction and when the two segments are divided (a-b), the result becomes strange 1 = 2 Because in the end, the value 1 = 2 appears (a contradiction with the comparison of the value of 2 numbers)	Explicit Controversial	Deliver that the process makes sense, but the results do not because obtained 1 = 2. The subjects are not able to find the main components that cause contradictions.

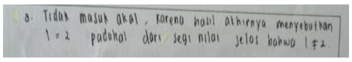
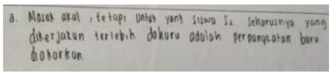
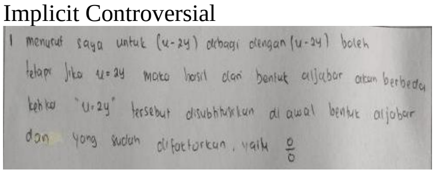
		 a. Tidak masuk akal, karena hasil akhirnya menyebutkan 1 = 2 padahal dari segi nilai jelas bahwa 1 ≠ 2 Copy of the subject's answer It does not make sense because the result says 1 = 2 even though in terms of value, it is clear that 1 ≠ 2 Interview S1: In my opinion, it is not right because one is not possible equals 2.	
3.	• Using procedures that are commonly done, i.e., do the work first and then take root and get it 2 • But a rank-up procedure that was reappointed also made sense; the result was -2. It is strange, but it makes sense.	Explicit Controversial  a. Masuk akal, tetapi untuk yang siswa S2 seharusnya yang dikerjakan terlebih dahulu adalah perpangkatan baru diakarkan Copy of the subject's answer It makes sense, but for Master's students, a new departure should be made first. Interview Q: Why is the appointment first then rooted? S1: Because it is easy	The subject felt the steps made sense, but the result was contradictory because 2 = -2 was obtained, but the subjects could not explain the components that caused contradictions.

Table 4.
Subjects' behaviors at exploration level

No	Subject Behavior	Answer of subjects	The Characteristics of Exploration Level
1.	Allow division with x-2y. The subject gives undefined information at the end of the work	Implicit Controversial  1 menurut saya untuk (x-2y) dibagi dengan (x-2y) boleh tetapi jika x=2y maka hasil dari bentuk aljabar akan berbeda ketika "x=2y" tersebut disubstitusikan di awal bentuk aljabar dan yang sudah difaktorkan, yaitu $\frac{0}{0}$ Copy of the subject's answer In my opinion, for (x-2y) divided by (x-2y) maybe, but if x = 2y, the results of the algebraic form will be different when the "x = 2y" is substituted at the beginning of the algebraic form and already factored in $\frac{0}{0.}$ Interview Q: In your answer, $\frac{0}{0}$appears. Why? S2: I mean that if divided by $\frac{x-2y}{x-2y}$, it is OK if the value is not $\frac{0}{0}$. For example, $\frac{0}{0}$ is undefined.	Capturing contradictions and tracing the components that cause contradictions, the subjects, gives an undefined explanation.

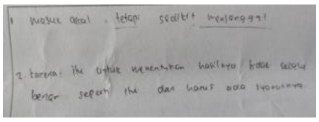
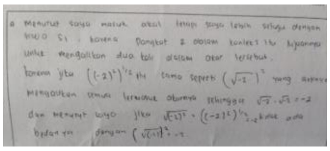
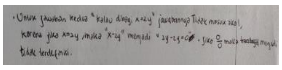
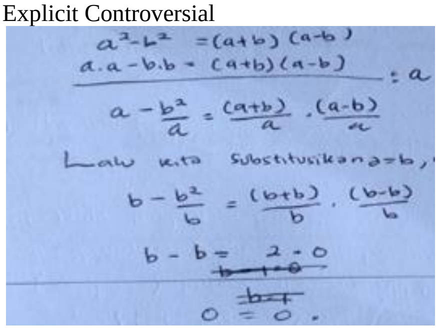
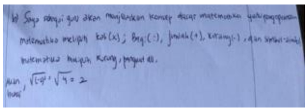
2.	Conveying that there is something strange Delivering that the results are incorrect and need to bring up conditions	Explicit Controversial  Copy of the subject's answer It makes sense, but it is a bit strange. Therefore, determining the result is not always true, and there must be a condition. Interview Q: How strangely do you mean? S2: The final answer is 1 = 2, that is the one, in my opinion, there must be a requirement in early on about that	The subject felt there was a contradiction and was able to bring up the component needs for conditions but could not produce the right solution.
3.	Complete the root and then go to the next step	Explicit Controversial  Copy of the subject's answer In my opinion, it makes sense, but I am more in agreement with S1 students because rank 2 in that context aims to multiply twice in the root. Therefore if $((-2)^2)^{\frac{1}{2}}$ is the same as $\left(\sqrt{-2}\right)^2$, which means to multiply everything, including the root, so $\sqrt{-2}\cdot\sqrt{-2} = -2$, and in my opinion, if $\sqrt{(-2)^2} = ((-2)^2)^{\frac{1}{2}} = -2$ is no different from $\left(\sqrt{-2}\right)^2 = -2$ Interview Q: What do you think about the process of the work on this problem? S2: Finish in the roots, and then you can go to the next step.	Students already feel the contradiction and show the process of work that was done first.

Table 5.
Subjects' behaviors at the clarification level

No	Subject Behavior	Answer of subjects	The Characteristics of Clarification Level
1.	Convey that if divided by x-2y, the result is undefined and should not be done Explain the basic concepts of rational functions to explain to students	Implicit Controversial  Copy of the subjects' answer For the second answer "if divided" the answer does not make sense, because if x = 2y, then "x = 2y" becomes "2y-2y = 0". If $\frac{0}{0}$ is undefined Interview Q: Why cannot x-2y divide it? S3: Because if it is done, undefined results will appear	Make solutions logically mathematically or explain various reasons that can be used to justify solutions to contradictions. This is revealed by the subjects in explaining the concept of undefined and giving rise to rational functions.

2.	Convey that the equation cannot be simplified. Bring up the concept of similarity when explaining to students	Explicit Controversial  $a^2-b^2=(a+b)(a-b)$ $a.a-b.b=(a+b)(a-b)$ $:a$ $a-\frac{b^2}{a}=\frac{(a+b)}{a}.\frac{(a-b)}{a}$ Law kita substitusikan a=b, $b-\frac{b^2}{b}=\frac{(b+b)}{b}.\frac{(b-b)}{b}$ $b-b=2.0$ $0=0.$ Interview Q: Why do you need to explain the concept in common with students? S: Because that is the initial foundation of whether a problem given can be simplified or not.	The subjects can make a solution by bringing up vague concepts and similarities.
3.	Convey that the order of operations needs to be considered	Explicit Controversial  Copy of the subject's answer b) As a teacher, I will explain the basic concepts of mathematics, namely the operation of mathematics including times (×); for (-); and mathematical symbols including parenthesis, rank, etc. It will be true $\sqrt{(-2)^2}=\sqrt{4}=2$ Interview Q: If you become a teacher, how will Do you explain this concept? S3: I will associate with the new term "sequence of operations" to this root concept or not.	The subjects can convey the sequence of operations and relate it to the concept of the previously presented sequence.

From the analysis and presentation of controversial reasoning, it is found that the characteristics of the level of controversial mathematical reasoning are shown in Table 6.

Table 6.
Characteristics of the level of controversial mathematical reasoning

Levels	Mathematical Controversial Reasoning
Initial	Students can recognize controversy (contradiction) but cannot explore the components of the problem that cause controversy (contradiction). For example, students recognized $\frac{0}{0}$, 1 = 2, 2 = -2 were controversial issues but were unable to trace which component caused the controversy.
Exploration	Students can recognize problems that cause controversy (contradiction) and trace the components that cause the problem to become controversial (contradiction). However, they were unable to clarify the problem as a solution. For example, students are able to explore $\frac{x-2y}{x-2y}$ with x = 2y; a (a-a) = (a + a) (a-a) the two sides are divided (a-a); operation of the square and the root respectively as components which caused controversy. However, they were unable to clarify these problems to come up with solutions.
Clarification	Students can clarify problems using reasons that make sense or come up with various reasons that can be used to justify solutions. For example, students can clarify and justify simplifying the algebraic form if $\frac{2x^2-3xy-2y^2}{x-2y}$ exists. In other words, the algebraic form is defined, meaning x ≠ 2y. In the root and exponential operations, students clarified that the sequence influences the roots and powers of even numbers. For odd roots and exponents, you do not need to pay attention to the order.

IV. CONCLUSION

Controversial reasoning is very important because it often occurs in everyday life, which is called general controversy. In the context of solving mathematical problems, this study finds the characteristics of three levels of controversial mathematical reasoning, namely initial, exploration, and clarification. The characteristics of the subjects indicate the initial level by recognizing the contradiction but not knowing the components causing the contradiction. In problem 1, the subject recognizes that when x = 2y, it results in x-2y = 0 and controversy when simplifying $\frac{x-2y}{x-2y}$). In problem 2, the subjects felt the process made sense, but the result was contradictory 1 = 2. In problem 3, the subjects recognized the process as reasonable, but the result was a contradiction, 2 = -2. Furthermore, when asked to provide reasons for the subject, they were unable to explain properly.

The exploration level is indicated by the characteristics of the subjects being able to recognize contradictions and explore the components of the problem that cause contradictions. In problem 1, the subject explores the simplification of the form $\frac{x-2y}{x-2y}$; when x = 2y, the form becomes 0/0, undefined. The subjects explain that there should be a condition x ≠ 2y, but there is no condition for this problem. In problem 2, the subject explores the factoring steps there is no problem a2 - a2 = (a-a) (a + a). Nevertheless, at the time of simplification, where the two sides are divided (a-a), his process becomes a problem. In problem 3, the subjects explore the properties of roots and squares. When the squares take precedence, there is no problem, but it becomes a problem when the roots are changed to powers of 1/2.

The clarification level is indicated by the characteristics of the subjects being able to clarify the existence of contradictions and make mathematical, logical solutions or explain various reasons that can be used to justify the resolution of contradictions. The subject clarifies that the statement in problem one is simplifying; therefore, the simplified form must be defined. It must be x ≠ 2y. In problem 2, the subject clarified that dividing the two sides by (a-a) is wrong because it is undefined. This wrong step resulted in an incorrect result, 1 = 2. In problem 3, the subject clarified that the roots and exponents of even numbers are affected by the sequence of operations, but there will be no problem if the exponents and roots are odd numbers. The 185 subjects who answered consistently can be distributed based on the levels: initial 55 (29.73%), exploration 52 (28.11%), and clarification 78 (42.16%) subjects.

This study found that an in-depth study related to controversial reasoning and other learning models is needed. It can also be linked to students' creativity and critical thinking while solving controversial questions. This paper is a novel because it seeks to contribute to the current debate in the literature [21, 24, 26, 32] on controversial reasoning. The scientific novelty of the article also consists of large-scale studies conducted that describe the importance of controversial reasoning in both theory and practice in life.

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