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Spatial Reasoning Profile of The Students with Good Number Sense Ability

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Spatial Reasoning Profile of The Students with Good Number Sense Ability

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Abstract. Mathematics teacher must have a good basic mathematics comprehension such as number sense and spatial reasoning before teach it to the students. Those ability will help a person to solve mathematical problems effectively. The aim of this study is to describe how number sense and spatial reasoning was used to solve mathematical problems. This research is a qualitative-descriptive research with 27 of second year pre-service mathematics teachers as a participant. Subject were selected using purposive sampling by conducting number sense ability test. The one with good number senses abilities were given advanced test related to spatial reasoning. The result of this research shows that pre-service mathematics teacher with good number sense ability can solve the given problems correctly. Subjects can flexibly conduct mental calculation, mathematical computation, and create various number representation accurately and are able to create efficient problem-solving strategy by combining mental rotation and spatial visualization within the process.

1. Introduction

This century's mathematical learning is no longer fixated only to counting and calculating. Mathematics become very important for daily life, professional qualification and for higher educational level [1], [2]. The learning is more related to the implementation on the daily life, while in the daily life, someone can't be separated from mathematics (e.g. time, transactions, adding some seasoning to food and et cetera). Therefore, it becomes very important for someone to own a good mathematical ability. This should be started with understanding the numbers along with its operations, then utilizing intuition on implementing numbers to solve the problems found in the daily life flexibly and using the most efficient strategy. The intuition is often called number sense [3], [4].

The term number sense can be interpreted as one's understanding towards numbers and its operations along with the skill to utilize it into mental calculation, mathematical computation, and creation of various numbers representations and utilize it to solve daily life problems flexibly and efficiently [5], [6]. Another version describes number sense as an understanding and ability that makes someone efficiently utilize various correlations between numbers and its operations, use benchmarking to assess the amount of a number and can determine the irrational result [2], [7]. A lot of advantage are achieved by having a very good number sense ability, both in formal education and in daily life. [8]. In formal lesson, this will of course make a person to have a better mathematics result [9]. However, it

doesn't stop only at this ability. There are many mathematical problems that requires spatial reasoning as the decisive factor of learning successfulness [10].

Furthermore, mathematics is strongly correlated with spatial reasoning [11]. Teacher oversees spatial reasoning also as an important component on mathematical learning successfulness and to solve mathematical problems [10], [12]. The calculation in mathematics starts at early age and continues until someone's mature by representing numbers mentally in various spatial format (e.g. the mental number line) [11], [13]. So, spatial reasoning is required to understand mathematical symbols, especially the ones that have correlations with numbers, so that the mathematical learning achievement can be increased.

Based on the previous explanation, it can be said that the successfulness of learning mathematics strongly correlates with number sense and spatial reasoning ability. This is reinforced by Parviainen [1] that states that people were born with number sense and spatial sense, and mathematical skills starts early. As an addition, because of the reciprocal relationship between those two, someone will connect numerical skills, spatial skills and reasoning skills on learning mathematics [14], [15]. It is why, this research aims to describe how number sense and spatial reasoning are used by university students that have good number sense ability to solve mathematical problems.

2. Research Method

The research is a qualitative-descriptive research with 27 of second year pre-service mathematics teachers are involved as participants. Subjects are selected using purposive sampling method. The criterion is a student that has high category number sense. The data are collected by conducting number sense ability test, contextual problem test and interview. The number sense ability instrument consists of 10 questions that needs to be done in 15 minutes. Meanwhile, contextual problem test needs will be given 2 questions that needs to be done in 30 minutes of time.
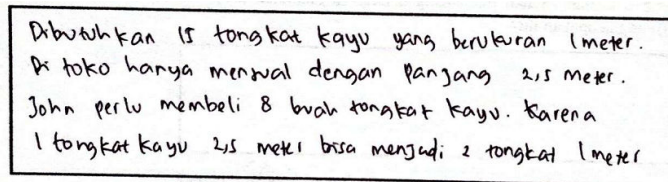
After all 27 participants finished the test, participants with good number sense ability will be chosen as subjects then interview step will take place. The interview process is conducted as an effort to confirm and to dig deeper information from the answers provided by the subjects. The categorization of number sense ability in this research refers to number sense ability test result where a score ≤ 6 is categorized as low, score between 6 and 8 is categorized as average, and score ≥ 8 is categorized as good [16]. Besides that, spatial reasoning that was mentioned in this research focuses on mental rotation and spatial visualization. Mental rotation means the 2D and 3D object rotation ability clockwise or counter-clockwise, while spatial visualization correlates with symmetry, pattern, 2D or 3D shapes and its correlations, reflections and symmetries of an object [17], [18].

3. Result and Discussion

The number sense ability significantly affects mathematical learning achievement [19]. This is why someone's number sense ability can be a benchmark whether the person will find difficulties or not upon solving mathematical problems. The number sense test result in this research shows that there are only 4 participants (14.81%) in good category, 7 participants (25.93%) in average category and 16 participants (59.26%) in low category. Based on the categorization result, one participant, AS from good category, was chosen to explain the mathematical problem solving that was done before. Both given problems aim to see how does the subject implement the owned number sense ability, especially to understand between context and necessary computation along with the correlation with spatial reasoning especially mental rotation and spatial visualization.

3.1. Mathematical problem 1

The first given problem is "John needs 15 wooden sticks, each 1 meter long for boy scout. Home depot only sell it each 2.5-meter-long. How many wooden sticks does John need?". This problem mainly correlated with someone's number sense ability on conducting estimation on calculating how many wooden sticks does John need. Subject AS solve problem number 1 with an accurate result. But the subject didn't write down the solution steps clearly. SR wrote down known information from the question along with the answer. The following Figure 1 is the answer provided by AS.

Translation: John needs 15 wooden sticks, each 1 meter long. Stores only sell it each 2.5-meter-long. John needs to buy 8 wooden sticks because 1 2.5-meter-long wooden stick can be turned into 2 1-meter-long wooden sticks.
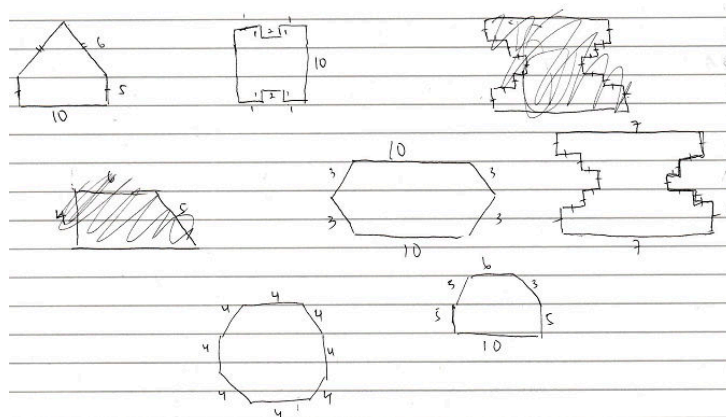
Fig. 1. Mathematical problem-solving number 1

Based on Figure 1, it can be seen that AS understand the given information, which is the need of 15 wooden sticks of 1-meter length. Meanwhile the home depot only sells wooden stick at 2.5-meter length. AS didn't write down what was the question on the problem on the answer sheet, but directly solve the problem by stating the answer, "John needs to buy 8 wooden sticks". On Figure 1, AS also wrote down the reasoning of the answer which is, "Because 1 2.5-meter-long wooden stick can be turned into 2 1-meter-long wooden sticks.". Then at the interview process, it is known that AS also understands that the problem is related to estimation computation. AS stated, that the question of the problem is the amount of wood that John needs to buy. So, with the information that has been given before, AS was picturing in his mind that from the 2.5-meter-long wood, it can be split into 2 pieces of wooden sticks each with 1 meter of length stick and 1 stick with 50cm of length. This process involved spatial reasoning in a form of spatial visualization ability. Through the spatial reasoning, AS can estimate the result because each 1 wooden stick of 2.5-meter length can only be split into 2 sticks, each with 1 meter of length. Furthermore, by doing mental calculation without pencil or paper as calculation tool, AS can directly divide 15 by 2 to get 7.5 as the result, which means only 7 wooden sticks only. Therefore, AS stated that the amount of the wooden sticks that John needed to buy is 8 sticks, by adding 1 extra stick to get the 15 wooden sticks result that were initially required.

Based on the explanation, it is known that subject AS was using spatial reasoning to help him collecting the problem-solving idea and to make him flexibly able to do mental calculation and using estimation computation well. Estimation computation as an important ability for someone to have in this century will be very useful to help someone to get a deeper understanding about number sense and arithmetical operation [20]. The existence of the ability can also help someone on calculating fast without the help of calculator or other calculation tools, even without having to initially find pencil and paper. Therefore, by having a good number sense ability, someone can also have self-confidence to think flexibly on solving various mathematical problems related to daily life with an efficient strategy [6], [9], [21]. As an addition, when someone learn to understand mathematical concept, it is actually for the sake of solving mathematical problem. Meanwhile, by solving mathematical problem, one can also develop other different skills at the same time [22].

3.2. Mathematical problem 2

The second problem that has to be solved by subject AS is, "A carpenter has a wooden block with a length of 32 meters. He wants to build a fence to block the plants. Create as many garden designs as you can under the condition: the fence needs should be buildable using 32 meters of wooden block!". Through this problem, subject is expected to be creative and using his number sense ability to create various number representation by also involving the corresponding illustrations as an efficient solution strategy. Figure 2 shows the mathematical problem solving that subject AS did.

Fig. 2. Mathematical problem-solving number 2

Based on Figure 2, it can be seen that the subject can create 6 garden design that fits the given condition. AS can utilize his number sense ability well, which is by creating various appropriate number representations. Besides, the number of designs that AS created shows that AS has an awareness of the existence of various strategies that can be implemented to solve the given mathematical problems. Based on the interview result, AS knows the given information, which is 32 meters wooden block to create fence for plants, accordingly to the designs presented. AS also stated that the problem requires subject to create various garden design that fits to the same condition, which is the fence should be created using the 32 meters wooden block. Then, AS tried to solve the problem by creating the garden designs. AS did finish 6 different designs by using different number representation by adjusting the side of each two-dimensional figure on each design. The basic thing on number sense is the reasoning ability such as verbal and symbolic ability to create connections among few representations or among mathematical ideas. [23], [24]. This process becomes very important to find a pattern and representation structure, accordingly to the new situation to facilitate the solutions that are still only exist in a form of abstract thoughts [25]. Therefore, number sense holds a very important role as a benchmarking of how far a person can utilize the owned number sense ability into various problem situations with numbers [26].
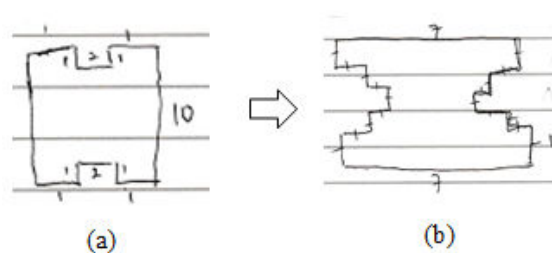
Fig. 3. Answer modification provided by the subject

Based on Figure 3, subject modified the answer that he got by rotating drawing (a) for 90° clockwise. To make the solution different from the previous one, he then formed and rearranged the 2D shape on drawing (a) by adding more sides. So that a new form is created. This process is still happening in their mental. Furthermore, by doing mental calculation, subject calculate the size of each side of the two-dimensional figure that he imagined where the sum of all sides equals 32 meters. After it felt right, subject visualize the problem solving by drawing on his answer sheet. The process describes that the subject fuse spatial reasoning and number sense ability that he owns upon solving the problem. Subject uses mental rotation and spatial visualization ability upon imagining the 2D image and then utilize number sense ability upon estimating the proper size for each side of the two-dimensional figure that he created and then proved using mental calculation
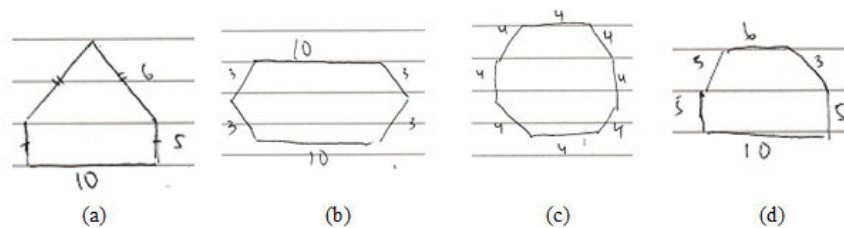
Fig. 4. Answer variance that the subject provided.

Figure 4 shows the answer variance provided by the subject. Through the interview, it is known that the subject decided first what two-dimensional figure that will be used to solve the problem then conduct mental calculations of numbers of sum of 32. This shows that the subject understands the correlation between problem context and the needed calculation, because that problem is related to 2D figure circumferences. This process involves number sense ability which is applying knowledge of facility with number and operations to computational settings. The next step is visualizing the problem solution into drawings by adjusting the size of each sides. The drawing activity involves the ability to visualize an object and reflect it by rotating and arrange it into other different shapes. This involves spatial reasoning which is mental rotation and spatial visualization. The series of processes that the subject has done upon creating a result as shown on Figure 4 shows that the student uses spatial reasoning and number sense at the same time to create the correct problem solving. Both are complementary to each other [1].

Generally, based on the two results, it is seen that number sense and spatial reasoning hold important role on mathematical problem solving [9] and both are correlated to each other [27]. Number sense is a cognitive process that will give logical description about the situation of a problem, developing strategies, creating mathematical reasoning, making decisions and paying attention to patterns between situations or concepts [28] by involving spatial reasoning upon representing numbers mentally [11], [13]. This research also shows that the subject can solve the two given mathematical problems correctly. This is corresponding with the previous research which mentioned that someone with a good number sense ability will also have a good management ability and good numbers intuition ability [26], [29].

4. Conclusion

Based on the explanation, it can be concluded that in a mathematical problem-solving process, pre-service mathematics teacher that has good number sense ability shows spatial reasoning and number sense ability at the same time. The involved spatial reasoning are mental rotation and visualization that are utilized to draw the problem situation upon deciding the problem solution ideas. Meanwhile, the number sense ability that is also involved are accurate calculation estimation, flexible on conducting mental calculation and sensitivity on finding various solution strategy that corresponds and efficient.

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